Also solved by Paul S. Bruckman, L. Kuipers, Bob Prielipp, Lawrence Somer, and the proposer.

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LETTER TO THE EDITOR

February 3, 1989

Dear Dr. Bergum,

I'd like to point out that some results which appeared in Michael Mays's recent article, "Iterating the Division Algorithm" [Fib. Quart. 25 (1987):204-213] were already known.

In particular, his Algorithm 6, which on input \((b, a)\) sets \(a = a \) and \(a \equiv b \mod a\), appeared in my paper, "Metric Theory of Pierce Expansions," [Fib. Quart. 24 (1986):22-40]. His Theorem 4, proving that \(L(b, a) \leq 2\sqrt{b} + 2\) [where \(L(b, a)\) is the least \(n\) such that \(a = 0\)], appears in my paper as Theorem 19.

Let \(\Omega, \Omega'\) be defined as follows: we write \(f(n) = \Omega(g(n))\) if there exist \(c, N\) such that \(f(n) \geq cg(n)\) for all \(n > N\). We write \(f(n) = \Omega'(g(n))\) if there exists \(c\) such that \(f(n) \geq cg(n)\) infinitely often. Since my paper appeared, I have proved

\[
\max_{1 \leq a \leq n} L(n, a) = \Omega'(\log n)
\]

and

\[
\sum_{1 \leq a \leq n} L(n, a) = \Omega(n \log \log n).
\]

The details are available to those interested.

Recently, I also stumbled across what may be the first reference to this type of algorithm. It is J. Binet, "Recherches sur la th\'eorie des nombres entiers et sur la r\'esolution de l'\'equation ind\'etermine du premier degr\'e qui n'admet que des solutions enti\'eres," J. Math. Pures Appl. 6 (1841):449-494. Binet's algorithm, however, takes the absolutely least residue at each step, rather than the positive residue, and it is therefore easier to prove there are no long expansions.

Sincerely yours,

Jeffrey Shallit