REPLY TO EXPLORING FIBONACCI MAGIC SQUARES*

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Problem. For \( n \geq 2 \), show that there do not exist any \( n \times n \) magic squares with distinct entries chosen from the set of Fibonacci numbers, \( u_1 = 1, u_2 = 2, u_{n+2} = u_{n+1} + u_n \) for \( n \geq 1 \).

Proof. Trivial for \( n = 2 \).

If an \( n \times n \) magic square existed for some \( n \geq 3 \) with distinct Fibonacci entries, then the requirement that the first three columns add to the same number would yield the equalities:

\[
(*) \quad F_{i_1} + F_{i_2} + \ldots + F_{i_n} = F_{j_1} + F_{j_2} + \ldots + F_{j_n} = F_{k_1} + F_{k_2} + \ldots + F_{k_n}
\]

Since the entries are distinct, we may assume without loss of generality that \( F_{i_1} > F_{i_2} > \ldots > F_{i_n}, F_{j_1} > F_{j_2} > \ldots > F_{j_n} \)

and

\( F_{k_1} > F_{k_2} > \ldots > F_{k_n} \).

Noting that the columns contain no common elements, and by rearrangement if necessary, we assume \( F_{i_1} > F_{j_1} > F_{k_1} \), again without losing generality; thus, \( F_{i_1} \geq F_{k_1} + 2 \).

Now

\[
F_{i_1} + F_{i_2} + \ldots + F_{i_n} > F_{k_1} + 2
\]

while

\[
F_{k_1} + F_{k_2} + \ldots + F_{k_n} \leq \sum_{i=1}^{k_1} F_i = F_{k_1} + 2 - 1
\]

This contradicts the equality postulated in \((*)\), and we conclude no magic squares in distinct Fibonacci numbers are possible.

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