

A STRIP METHOD OF SUMMING LINEAR FIBONACCI EXPRESSIONS

BROTHER U. ALFRED
St. Mary's College, California

Given a linear Fibonacci expression such as

$$\begin{aligned} &362880 F_{r+21} - 2177280 F_{r+19} + 5594400 F_{r+17} - 8013600 F_{r+15} \\ &+ 6972840 F_{r+13} - 3759840 F_{r+11} + 1225230 F_{r+9} - 223290 F_{r+7} \\ &+ 19171 F_{r+5} - 512 F_{r+3} + F_{r+1} \end{aligned}$$

we wish to express this, for example, as

$$A F_{r+11} + B F_{r+10}$$

The formulas for doing so are well known being

$$F_n = F_{k+1} F_{n-k} + F_k F_{n-k-1}$$

and

$$F_n = F_k F_{n+k+1} - F_{k+1} F_{n+k}$$

However, the direct process can be replaced by a strip method in which the given coefficients are arranged in descending order of F subscripts, one space being allowed for each subscript, even though certain subscripts may be missing in the given linear expression. This may be done conveniently on ruled paper, the strip employed having the same spacing in its rulings as the paper.

The strip consists of the Fibonacci numbers in descending order. To obtain the coefficient of the higher subscript Fibonacci number in the summation, place the 1 above the zero at the place of the higher subscript, multiply each number on the strip by the corresponding given coefficient and add the results. To find the coefficient of the lower subscript Fibonacci number, do likewise with the 1 below the zero opposite the position of the lower subscript Fibonacci number.

The work is shown for the example given at the beginning of this note.

UPPER SUBSCRIPT CALCULATION

STRIP	GIVEN COEFFICIENTS	PRODUCTS
89	362880	32296320
55		
34	-2177280	-74027520
21		
13	5594400	72727200
8		
5	-8013600	-40068000
3		
2	6972840	13945680
1		
1	-3759840 (F_{r+11})	-3759840
0		
1	1225230	1225230
-1		
2	-223290	-446580
-3		
5	19171	95855
-8		
13	-512	-6656
-21		
34	1	34
		<hr/> 1981723

LOWER SUBSCRIPT CALCULATIONS

STRIP	GIVEN COEFFICIENTS	PRODUCTS
55	362880	19958400
34		
21	-2177280	-45722880
13		
8	5594400	44755200
5		
3	-8013600	-24040800
2		
1	6972840	6972840
1		
0	-3759840 (F_{r+10})	0
1		
-1	1225230	-1225230
2		
-3	-223290	669870
5		
-8	19171	-153368
13		
-21	-512	10752
34		
-55	1	-55
		<hr/> 1224729

The final result would thus be

$$1981723 F_{r+11} + 1224729 F_{r+10}$$

In carrying out these calculations it goes without saying that the products need not be written out but may be cumulated on a calculator.

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THE FIBONACCI ASSOCIATION ANNOUNCES.

The appearance of a booklet entitled: "Introduction to Fibonacci Discovery" by Brother U. Alfred, Managing Editor of the Fibonacci Quarterly. As the title implies the aim of this publication is to provide the reader with the opportunity to work out various facets of the Fibonacci numbers by himself. At the same time, there is sufficient help in the form of answers and explanations to reassure him regarding the correctness of his work.

The treatment is relatively brief, there being some sixty pages in all. The material was set up by typewriter and subsequently lithographed. The books have a paper cover and are held together by glue binding. Price per copy is \$1.50 with a quantity price of \$1.25 when four or more copies are ordered at once. The following topics are treated:

Discovering Fibonacci Formulas
 Proof of Formulas by Mathematical Induction
 The Fibonacci Shift Formulas
 Explicit Formulas for the Fibonacci and Lucas Sequences
 Division Properties of Fibonacci Numbers
 General Fibonacci Sequences
 The Associated "Lucas" Sequence
 The Fibonacci Sequence and Pascal's Triangle
 The Golden Section
 Matrices and Fibonacci Numbers
 Continued Fractions and Fibonacci Numbers

This booklet should provide the means of becoming acquainted with Fibonacci numbers and some of their main ramifications. It should serve as a useful reference for readers of the Fibonacci Quarterly who wish to learn about the main aspects of Fibonacci numbers. It should also prove of value to groups of competent high school or college students. While not recommended for the "pro", it might be a useful reference to have on hand to loan to students or fellow faculty members who want to know something about Fibonacci numbers.

The booklet is now available for purchase. Send all orders to:
 Brother U. Alfred, Managing Editor, St. Mary's College, Calif.
 (Note. This address is sufficient, since St. Mary's College is a post office.)