

6. Ch. A. Charalambides & M. Koutras. "On the Differences of the Generalized Factorials at an Arbitrary Point and Their Combinatorial Applications." *Discrete Math.* **47** (1983):183-201.
7. F. T. Howard. "Degenerate Weighted Stirling Numbers." *Discrete Math.* **57** (1985):45-58.
8. M. Koutras. "Non-Central Stirling Numbers and Some Applications." *Discrete Math.* **42** (1982):73-89.
9. N. E. Nörlund. *Vorlesungen über Differenzenrechnung*. Berlin: Springer, 1924.
10. J. Riordan. *An Introduction to Combinatorial Analysis*. New York: Wiley, 1958.

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GENERALIZED PASCAL TRIANGLES AND PYRAMIDS THEIR FRACTALS, GRAPHS, AND APPLICATIONS

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As stated by the author in his preface, this monograph is devoted to the more profound questions connected with the study of the Pascal triangle, and its planar as well as spatial analogs. It also contains an extensive discussion of the divisibility of the binomial, trinomial, and multinomial coefficients by a prime p , as well as the distributions of these coefficients with respect to the modulus p , or p^s , in corresponding arithmetic triangles, pyramids and hyperpyramids. Particular attention is also given to the subject of fractals obtained from the Pascal triangle and other arithmetic triangles. The author also constructs and investigates matrices and determinants whose elements may be binomial, generalized binomial and trinomial coefficients, and other special values. Furthermore, the author pays particular attention to the development of effective combinatorial methods and algorithms for the construction of basis systems of polynomial solutions of partial differential equations, including equations of higher order and with mixed derivatives. The algorithms he proposes are invariant with respect to the order, and the iteration, of operators arising in connection with the differential equations. Finally, the author also discusses non-orthogonal polynomials of binomial type, and polynomials whose coefficients may be Fibonacci, Lucas, Catalan, and other special numbers.

The monograph first published in Russia in 1990 consists of seven chapters, a list of 406 references, an appendix with another 126 references, many illustrations and specific examples. Fundamental results in the book are formulated as theorems and algorithms or as equations and formulas.

The intention of the translator is to make the work of Dr. Bondarenko widely accessible because he feels that Dr. Bondarenko has done the mathematical community a valuable service by writing a useful and interesting compendium of results on Pascal's triangle as well as its ramifications.

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