A VARIATION ON THE TWO-DIGIT KAPREKAR ROUTINE

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(Submitted June 1991)

In 1949 the Indian mathematician D. R. Kaprekar discovered a curious relationship between the number 6174 and other 4-digit numbers. For any 4-digit number n, whose digits are not all the same, let n' and n'' be the numbers formed by arranging the digits of n in descending and ascending order, respectively. Find the difference of these two numbers: $T(n) = n' - n''$. Repeat this process, known as the *Kaprekar routine*, on $T(n)$. In 7 or fewer steps, the number 6174 will occur. Moreover, 6174 is invariant; that is, $T(6174) = 6174$.

In the literature it is common to generalize the Kaprekar routine and apply it to any k-digit number in base g. Since there are only a finite number of k-digit numbers, repeated applications of T always become periodic. The result is not necessarily a single invariant; more frequently one or more cycles occur. The characterization of such cycles is a difficult problem which has not been completely solved. Among the questions studied are the following: Given k, for what value(s) of g does the Kaprekar routine produce a single invariant? When nontrivial cycles arise for a given g and k, how many cycles are there and what are their lengths? This author, among others, has studied these problems as well as many other fascinating questions associated with the above procedure. (See [1]-[12].)

Recently I was describing the Kaprekar routine to faculty colleagues. To demonstrate that not all k-digit numbers in base 10 give rise to a constant, I chose to illustrate the routine for 2-digit numbers. In that case, either one or two applications of T yields one of the numbers in the cycle

$$63 \to 27 \to 45 \to 09 \to 81 \to 63.$$

Embarrassingly, I made an arithmetic mistake, writing $T(96) = 96 - 69 = 37$ instead of $T(96) = 27$. Arleigh Bell, Associate Professor of Economics, asked what would happen if 10 or any other number r were always added to $T(n)$. What would the cycles look like in that case? Could there be a Kaprekar constant for some number r? This paper is an answer to his questions.

As is the usual practice, we will consider Bell's questions for a general base g. We will represent a 2-digit, base g number $n = a'g + a$, $0 \le a'$, $a < g$, by $n = [a' \;\; a]$. The Bell modification of the Kaprekar routine is a function $K_{[r' \;\; r]}(n)$ defined in the following manner. Let $[r' \;\; r]$ be a fixed 2-digit, base g number less than $[1 \;\; g-1]$; that is, $r' = 0$ or 1, $0 \le r \le g-1$ if $r' = 0$, and $0 \le r \le g-2$ if $r' = 1$. Then, for $n = [a' \;\; a]$

$$K_{[r' \;\; r]}(n) = |[a' \;\; a] - [a \;\; a']| + [r' \;\; r].$$

When the context is clear, we will omit the subscript and simply write $K(n)$. To see why we require $[r' \;\; r] < [1 \;\; g-1]$, note that

$$|[a' \;\; a] - [a \;\; a']| = [|a' - a| - 1 \;\; g - |a' - a|].$$

Now $|a' - a| - 1 \le g - 2$, so $|[a' \;\; a] - [a \;\; a']| \le [g-2 \;\; 1]$. Thus, the restriction $[r' \;\; r] < [1 \;\; g-1]$ insures that $K(n)$ is a 2-digit number.

Since there are only a finite number of 2-digit, base g numbers, the sequence

$$n, \; K(n), \; K^2(n), \; K^3(n), \ldots$$

must eventually repeat. If, for a given n, $K^i(n)=n$, where i is as small as possible, then we say that n is in a cycle of length i. We will denote a K-cycle by $\langle n_1, n_2, \ldots, n_i\rangle$, where $n_{j+1}=K(n_j)$ for $1\le j\le i-1$ and $n_1=K(n_i)$ We wish to characterize those n which are in cycles and to determine the lengths of these cycles.

For $n=[a' \ \ a]$, $d=|a'-a|$ is called the digit difference of n. Observe that, if $0<d$, then

$$|[a' \ \ a]-[a \ \ a']|=[d-1 \ \ g-d].$$

Thus, if $n=[a' \ \ a]$, $m=[b' \ \ b]$, and $d=|a'-a|=|b'-b|$, then $K(n)=K(m)$. In particular, if n is a 2-digit number whose digit difference is d, $K(n)$ equals

$$(1)\qquad \begin{array}{lll} [d & r-d] & \text{if } r'=0 \text{ and } 0\le d\le r \\ [d-1 & g-(d-r)] & \text{if } r'=0 \text{ and } r<d<g \\ [d+1 & r-d] & \text{if } r'=1 \text{ and } 0\le d\le r \\ [d & g-(d-r)] & \text{if } r'=1 \text{ and } r<d<g. \end{array}$$

Using (1), it is easy to see that the digit difference of $K(n)$ is

$$(2)\qquad \begin{array}{ll} |r-2d| & \text{if } r'=0 \text{ and } 0\le d\le r \\ |g+r+1-2d| & \text{if } r'=0 \text{ and } r<d<g \\ |r-1-2d| & \text{if } r'=1 \text{ and } 0\le d\le r \\ |g+r-2d| & \text{if } r'=1 \text{ and } r<d<g. \end{array}$$

We will denote the digit difference of $K_{[r' \ r]}(n)$ in (2) by $D_{[r' \ r]}(d)$ or $D(d)$. Note that, if $K([a' \ \ a])=[b' \ \ b]$, then $D(|a'-a|)=|b'-b|$. Thus, each $K_{[r' \ r]}$-cycle gives rise to a $D_{[r' \ r]}$-cycle of the same length. If we can characterize the D-cycles, then we will have made substantial progress in characterizing the K-cycles. That is, we will know how many such cycles there are and the length of each one.

As an example, let $g=10$, $r'=0$, and $r=7$. That is, we wish to apply the routine to base 10 numbers with 7, the added term. Using (2), we find

$$\begin{array}{lllll} D(0)=7 & D(1)=5 & D(2)=3 & D(3)=1 & D(4)=1 \\ D(5)=3 & D(6)=5 & D(7)=7 & D(8)=2 & D(9)=0. \end{array}$$

Thus, the D_7-cycles are $\langle 1,5,3\rangle$ and $\langle 7\rangle$. From these, it is easy to determine that the K_7-cycles are $\langle 34,16,52\rangle$ and $\langle 70\rangle$.

Examination of (2) shows that $D(d)$ always has the form $|s-2d|$ for some s. Consequently, we will first study a function based on this observation. In particular, let s be a fixed positive integer. For d with $0\le d\le s$, define $F_s(d)=|s-2d|$. Since $0\le F_s(d)\le s$, cycles must occur. The following observations about F, collected in a single theorem for convenience, are obvious.

Theorem 1: Let s and i be positive integers and let d be an integer satisfying $0\le d\le s$. Then

- ***(a)*** $F_s(s)=s$, so $\langle s\rangle$ is an F_s-cycle of length 1.
- ***(b)*** $F_{is}(id)=iF_s(d)$
- ***(c)*** d is in an F_s-cycle if and only if id is in an F_{is}-cycle. In particular, $\langle d_1, d_2, \ldots, d_n\rangle$ is an F_s-cycle if and only if $\langle id_1, id_2, \ldots, id_n\rangle$ is an F_{is}-cycle.
- ***(d)*** $F_s^i(d)$ is congruent to either $2^i d$ or $-2^i d$ modulo s. □

For convenience, we will use the notation $F_s^i(d) \equiv \pm 2^i d \pmod{s}$ to represent statement (d) in Theorem 1.

Theorem 2: Suppose $2^k \| s$. Let d be an integer satisfying $0 \le d \le s$. If d is in an F_s-cycle, then $2^k \| d$.

Proof: Since $2^k \| s$, $s = 2^k t$ where $0 \le k$ and t is an odd positive integer. Write $d = 2^i w$ with $0 \le i$ and w odd. Then $F(d) = F(2^i w) = |2^k t - 2^{i+1} w|$. So

$$\begin{array}{ll} 2^{i+1} \| F(d) & \text{if } 0 \le i < k-1 \\ 2^{k+1} | F(d) & \text{if } i = k-1 \\ 2^k \| F(d) & \text{if } i > k-1. \end{array}$$

Thus, $2^j | F^j(d)$ for $j \le k-1$ and $2^k \| F^j(d)$ for $k+1 \le j$. Consequently, d is in a cycle only if $2^k \| d$. □

Corollary 1: Suppose $2^k \| s$. Let d be an integer satisfying $0 \le d \le s$. Then d is in an F_s-cycle if and only if $2^k \| d$.

Proof: First, suppose s is odd. By Theorem 2, it is sufficient to show that if d is odd, then it is in a cycle. Since s and d are odd, $(s+d)/2$ and $(s-d)/2$ are both nonnegative integers less than or equal to s. One of these numbers is odd and the other is even. Moreover, $F((s+d)/2) = d$ and $F((s-d)/2) = d$. Consequently, d has an odd predecessor. Since this is true for all odd integers between 0 and s, d must be in a cycle.

The case when s is even follows immediately using Theorem 2 and Theorem 1(c). □

Corollary 2: An integer s has only one F_s-cycle, namely $\langle s \rangle$, if and only if $s = 2^k$ for some k.

Proof: The proof is immediate using Theorem 1(a) and Corollary 1. □

By the results above, to characterize F-cycles it is sufficient to determine cycles for odd s. Additionally, we need only consider those d which are odd, have $\gcd(d, s) = 1$ and satisfy $1 \le d \le s-2$. We will call cycles containing such d nontrivial. All other cycles are trivial since they may be obtained using (a) and (c) of Theorem 1.

We will illustrate the comments above by finding the F-cycles for $s = 33$. By Corollary 1, only odd integers are in a cycle. Nontrivial F-cycles for $s = 3$ and $s = 11$ are $\langle 1 \rangle$ and $\langle 1, 9, 7, 3, 5 \rangle$, respectively. Thus, by Theorem 1(c),

$$\langle 11 \rangle,\ \langle 3, 27, 21, 9, 15 \rangle,\ \langle 33 \rangle \tag{3}$$

are trivial F-cycles for $s = 33$. We now want to calculate the nontrivial F-cycles. An efficient method, described for the general case and then applied to $s = 33$, is as follows. By Theorem 1(d), $F(d)$ is congruent to either $2d$ or $-2d$ modulo s. For s and d odd, exactly one of the numbers $2d$ or $-2d$ is congruent modulo s to an odd positive integer less than s. So to compute the cycle containing d, it is sufficient to calculate $\pm 2F^i(d) \equiv \pm 2F(F^{i-1}(d))$, choosing the appropriate sign so that the result modulo s is an odd integer. Applying this to our example $s = 33$ with $d = 1$ gives $1, -2 \equiv 31,\ 62 \equiv 29,\ 58 \equiv 25,\ 50 \equiv 17,\ 34 \equiv 1$, which yields the F-cycle

$$\langle 1, 31, 29, 25, 17 \rangle. \tag{4}$$

At this point we check to see if all odd integers d, $1 \le d \le s$, are accounted for. If not, we repeat the above procedure. In the present example, $d = 5$ is not contained in any of the cycles in (3) or (4). So we consider $5,\ -10 \equiv 23,\ 46 \equiv 13,\ -26 \equiv 7,\ -14 \equiv 19,\ 38 \equiv 5$, which gives

$$\langle 5, 23, 13, 7, 19 \rangle. \tag{5}$$

Thus, there are five F_{33}-cycles which are given in (3), (4), and (5).

For future reference, we record the F_s-cycles for $0 \le s \le 15$:

s	F_s - cycles	s	F_s - cycles
0	$\langle 0 \rangle$	8	$\langle 8 \rangle$
1	$\langle 1 \rangle$	9	$\langle 1,7,5 \rangle$, $\langle 3 \rangle$, $\langle 9 \rangle$
2	$\langle 2 \rangle$	10	$\langle 2,6 \rangle$, $\langle 10 \rangle$
3	$\langle 1 \rangle$, $\langle 3 \rangle$	11	$\langle 1,9,7,3,5 \rangle$, $\langle 11 \rangle$
4	$\langle 4 \rangle$	12	$\langle 4 \rangle$, $\langle 12 \rangle$
5	$\langle 1,3 \rangle$, $\langle 5 \rangle$	13	$\langle 1,11,9,5,3,7 \rangle$, $\langle 13 \rangle$
6	$\langle 2 \rangle$, $\langle 6 \rangle$	14	$\langle 2,10,6 \rangle$, $\langle 14 \rangle$
7	$\langle 1,5,3 \rangle$, $\langle 7 \rangle$	15	$\langle 1,13,11,7 \rangle$, $\langle 3,9 \rangle$, $\langle 5 \rangle$, $\langle 15 \rangle$

Theorem 3: Let s be an odd positive integer and let m be the smallest integer such that $2^m \equiv \pm 1 \pmod{s}$. Then each nontrivial F_s-cycle is of length m and there are $\phi(s)/2m$ such cycles, where $\phi(s)$ is the Euler phi function.

Proof: As before, we write ± 1 to indicate that 2^m is congruent modulo s to either 1 or -1. Suppose d is odd with $\gcd(d, s) = 1$ and i is the smallest integer such that $F^i(d) = d$. That is, we assume that d is a nontrivial cycle of length i. By Theorem 1(d), $F^i(d) \equiv \pm 2^i d \pmod{s}$, so $\pm 2^i d \equiv d \pmod{s}$. Since $\gcd(d, s) = 1$, $2^i \equiv \pm 1 \pmod{s}$. Consequently, each cycle has length $i = m$. There are $\phi(s)/2$ odd positive integers less than s which are relatively prime to s. Therefore, there are $\phi(s)/2m$ nontrivial F-cycles. □

The smallest positive integer k such that $2^k \equiv 1 \pmod{s}$ is called the order of 2 modulo s and is denoted by $\mathrm{ord}_s 2$.

Corollary 3: Let s be an odd positive integer and let m be the smallest integer such that $2^m \equiv \pm 1 \pmod{s}$. If $2^m \equiv +1 \pmod{s}$, then each nontrivial F_s-cycle has length equal to $\mathrm{ord}_s 2$; otherwise, the length equals $(\mathrm{ord}_s 2)/2$.

Proof: If $2^m \equiv +1 \pmod{s}$, then $\mathrm{ord}_s 2 = m$ and the result follows immediately from Theorem 3.

If $2^m \equiv -1 \pmod{s}$, then $2^{2m} \equiv +1 \pmod{s}$. By a well-known theorem from Number Theory, $k | 2m$ where $k = \mathrm{ord}_2 2$. If k is odd, then $k | m$ and $m = kq$ for some q. But this implies that $2^m \equiv (2^k)^q \equiv 1 \pmod{s}$, which is a contradiction. Thus, it must be the case that k is even and $(k/2) | m$. If $(k/2) < m$, then $m = (k/2)q$ with $1 < q$. But then $2^{(k/2)2} \equiv 1 \pmod{s}$, which contradicts the choice of m. Thus, $m = k/2 = (\mathrm{ord}_s 2)/2$. □

Corollary 4: Let p be an odd prime. Then the length of each nontrivial F_p-cycle equals

$$m = \mathrm{ord}_p 2 / \gcd(\mathrm{ord}_p 2, 2).$$

Proof: Let m be the smallest integer such that $2^m \equiv \pm 1 \pmod{p}$. The proof of Corollary 3 shows that if $2^m \equiv -1 \pmod{p}$, then $\mathrm{ord}_p 2$ is even and $m = \mathrm{ord}_p 2/2 = \mathrm{ord}_p 2 / \gcd(\mathrm{ord}_p 2, 2)$.

If $2^m \equiv 1 \pmod{p}$ with $m = \text{ord}_p 2$, then m must be odd. For if m were even, then $(2^{m/2})^2 \equiv 1 \pmod{p}$. Since p is prime, $2^{m/2} \equiv \pm 1 \pmod{p}$, which is a contradiction to the choice of m. Thus, $m = \text{ord}_p 2 = \text{ord}_p 2 / \gcd(\text{ord}_p 2, 2)$. □

Corollary 5: Let s be an odd positive integer and suppose 2 is a primitive root of s. Then s has only one nontrivial F_s-cycle.

Proof: Since 2 is a primitive root of s, $\text{ord}_s 2 = \phi(s)$. Moreover, there exists a unique positive integer i less than $\phi(s)$ such that $2^i \equiv -1 \pmod{s}$. By Corollary 3, the length of each nontrivial cycle is $\phi(s)/2$. Consequently, by Theorem 3, there is only one such cycle. □

We now state and prove three technical lemmas which will be useful when we apply this work to D-cycles.

Lemma 1: Let $s = g + r + 1$ and d be an integer satisfying $r < d < g$ and $r < F(d) < g$. Then $r < g/2$.

Proof: Suppose, to the contrary, that $g/2 \le r$. Since, by assumption, $r < d$, $g/2 < d$, which implies $g + r + 1 - 2d \le r$. Also, $d < g \le g/2 + r$ so that $2d - (g + r + 1) < r$. Thus,

$$F_s(d) = |g + r + 1 - 2d| \le r,$$

which is a contradiction to the hypothesis. □

Lemma 2: Let $s = g + r$ and d be an integer satisfying $r \le d < g$ and $r < F(d) < g$. Then $r < g/2$.

Proof: The proof is similar to that of Lemma 1. □

Lemma 3: Let $s = g + r$. If r has a predecessor under F_s, then $2|g$.

Proof: Suppose there exists d such that $F_s(d) = r$. Then either $g + r - 2d = r$ or $2d - (g + r) = r$. So either d equals $g/2$ or $r + g/2$. In either case, $2|g$. □

We are now in a position to characterize D-cycles.

Theorem 4: Let g be a positive integer and r an integer satisfying $0 \le r \le g - 1$. All F_r-cycles will be $D_{[0\ r]}$-cycles. If $r < g/2$ and there exists a d such that $r < F^i_{g+r+1}(d) < g$ for $0 \le i$, then this F_{g+r+1}-cycle is also a $D_{[0\ r]}$-cycle.

Proof: Since the added term is $[0\ r]$, the first two lines of (2) apply. From the first line, we see that all F_r-cycles will be $D_{[0\ r]}$-cycles. In order for the second line to give $D_{[0\ r]}$-cycles, it must be the case that all d in an F_{g+r+1}-cycle satisfy $r < d < g$. By Lemma 1, such cycles can occur only when $r < g/2$. □

As a consequence of Theorem 4, in order to find all $D_{[0\ r]}$-cycles for a given g, it is sufficient to examine all F_s-cycles for $0 \le s \le g + [(g+1)/2]$. For example, using (6), it is easy to find the $D_{[0\ r]}$-cycles for $g = 10$. These, as well as the corresponding $K_{[0\ r]}$-cycles, are:

(7)

r	$g+r+1$	$D_{[0\ r]}$-cycles	$K_{[0\ r]}$-cycles
0	11	$\langle 0\rangle$, $\langle 1,9,7,3,5\rangle$	$\langle 0\rangle$, $\langle 45,9,81,63,27\rangle$
1	12	$\langle 1\rangle$, $\langle 4\rangle$	$\langle 10\rangle$, $\langle 37\rangle$
2	13	$\langle 2\rangle$	$\langle 20\rangle$
3	14	$\langle 1\rangle$, $\langle 3\rangle$	$\langle 12\rangle$, $\langle 30\rangle$
4	15	$\langle 4\rangle$, $\langle 5\rangle$	$\langle 40\rangle$, $\langle 49\rangle$
5		$\langle 1,3\rangle$, $\langle 5\rangle$	$\langle 32,14\rangle$, $\langle 50\rangle$
6		$\langle 2\rangle$, $\langle 6\rangle$	$\langle 24\rangle$, $\langle 60\rangle$
7		$\langle 1,5,3\rangle$, $\langle 7\rangle$	$\langle 34,16,52\rangle$, $\langle 70\rangle$
8		$\langle 8\rangle$	$\langle 80\rangle$
9		$\langle 1,7,5\rangle$, $\langle 3\rangle$, $\langle 9\rangle$	$\langle 54,18,72\rangle$, $\langle 36\rangle$, $\langle 90\rangle$

Theorem 5: Let g be a positive integer and let r be an integer satisfying $0 \le r \le g-2$. All F_{r-1}-cycles will be $D_{[1\ r]}$-cycles. If $r < g/2$ and there exists a d such that $r < F^i_{g+r}(d) < g$ for $0 \le i$, then this F_{g+r}-cycle is also a $D_{[1\ r]}$-cycle. If $2|g$, and if $F^j_{g+r}(r+1) = r$ for some j, then r is in a $D_{[1\ r]}$-cycle.

Proof: Since the added term is $[1\ r]$, the third and fourth lines of (2) apply. From the third, we see that all F_{r-1}-cycles will be $D_{[1\ r]}$-cycles. In order for the fourth to give $D_{[1\ r]}$-cycles, it must be the case that all d in an F_{g+r}-cycle satisfy $r < d < g$. By Lemma 2, cycles such as these can occur only when $r < g/2$. There is one more way in which $D_{[1\ r]}$-cycles can arise. Note that $D_{[1\ r]}(r) = r+1$ and $D^i_{[1\ r]}(r) = F^{i-1}_{g+r}(r+1)$ for $2 \le i$. So if, for some j, $F^j_{g+r}(r+1) = r$, then r will be in a $D_{[1\ r]}$-cycle even though it may not be in an F_{g+r}-cycle. By Lemma 3, in order for r to have an F_{g+r} predecessor, g must be even. □

Finding $D_{[1\ r]}$-cycles which do not contain r is similar to finding $D_{[0\ r]}$-cycles. In particular, we examing F_s-cycles for $1 \le s \le g-3$ and $g \le s \le g+[(g-1)/2]$. For example, again using (6), it is easy to find these cycles for $g = 10$.

(8)

r	$r-1$	$g+r$	$D_{[1\ r]}$-cycles	$K_{[1\ r]}$-cycles
0		10	$\langle 2,6\rangle$	$\langle 64,28\rangle$
1	0	11	$\langle 0\rangle$	$\langle 11\rangle$
2	1	12	$\langle 1\rangle$, $\langle 4\rangle$	$\langle 21\rangle$, $\langle 48\rangle$
3	2	13	$\langle 2\rangle$	$\langle 31\rangle$
4	3	14	$\langle 1\rangle$, $\langle 3\rangle$	$\langle 23\rangle$, $\langle 41\rangle$
5	4		$\langle 4\rangle$	$\langle 51\rangle$
6	5		$\langle 1,3\rangle$, $\langle 5\rangle$	$\langle 43,25\rangle$, $\langle 61\rangle$
7	6		$\langle 2\rangle$, $\langle 6\rangle$	$\langle 35\rangle$, $\langle 71\rangle$
8	7		$\langle 1,5,3\rangle$, $\langle 7\rangle$	$\langle 45,27,63\rangle$, $\langle 81\rangle$

Missing from (8) are those $D_{[1\ r]}$-cycles which contain r. The final theorems address this special case.

Theorem 6: Let g be an even positive integer. When r equals 1, $g/2-2$ or $g/2-1$, then

$$(9) \qquad \begin{array}{l} \langle 2, g-3, \ldots, F^i_{g+1}(1), \ldots, 1\rangle \text{ with } 2 \le i \\ \langle g/2-2, g/2-1, g/2\rangle \\ \langle g/2-1, g/2\rangle \end{array}$$

are $D_{[1\ r]}$-cycles, respectively.

Proof: The last two cases are easily verified. For the first, by Corollary 1, 1 is in an F_{g+1}-cycle; in particular

$$\langle 1, g-1, g-3, \ldots, F^i_{g+1}(1), \ldots, 1\rangle.$$

Since $D_{[1\ r]}(1) = 2$ and $D_{[1\ r]}(2) = g-3$, applying the $D_{[1\ r]}$-algorithm gives

$$\langle 2, g-3, \ldots, F^i_{g+1}(1), \ldots, 1\rangle. \quad \square$$

Theorem 7: Let g and r be positive integers. If r is in an $D_{[1\ r]}$-cycle different from those in (9), then $r \le g/4-1$.

Proof: By Theorem 5, since r is in an $D_{[1\ r]}$-cycle, $F^j_{g+r}(r+1) = r$ for some $0 < j$. If $j = 1$, then $r = g/2-$, contradicting the hypothesis. Thus, $2 \le j$. Now,

$$D^3_{[1\ r]}(r) = F^2_{g+r}(r+1) = F_{g+r}(g-r-2) = |g-3r-4|.$$

By Lemma 2 and Theorem 6, $1 < r < g/2-2$ so that $D^3_{[1\ r]}(r) = g-3r-4$. If r is in an $D_{[1\ r]}$-cycle, then $r \le D^3_{[1\ r]}(r)$. This implies $r \le g/4-1$. $\square$

For $g = 10$, by Theorem 6, the following $D_{[1\ r]}$-cycles may be added to the list in (8):

(10)

r	$D_{[1\ r]}$-cycles	$K_{[1\ r]}$-cycles
1	$\langle 1, 2, 7, 3, 5\rangle$	$\langle 56, 20, 29, 74, 38\rangle$
3	$\langle 3, 4, 5\rangle$	$\langle 58, 40, 49\rangle$
4	$\langle 4, 5\rangle$	$\langle 59, 50\rangle$

By Theorem 7, these are the only $D_{[1\ r]}$-cycles that contain r. Thus, (7), (8), and (10) comprise a complete list of all $D_{[r',\ r]}$-cycles for $g = 10$.

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AMS Classification number: 11A99

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LOCAL INFORMATION

For information on local housing, food, local tours, etc. please contact:

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Call for Papers

Papers on all branches of mathematics and science related to the Fibonacci numbers as well as recurrences and their generalizations are welcome. Abstracts are to be submitted by March 15, 1994. Manuscripts are due by May 30, 1994. Abstracts and manuscripts should be sent in duplicate following the guidelines for submission of articles found on the inside front cover of any recent issue of *The Fibonacci Quarterly* to:

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