REDUCED AND AUGMENTED AMICABLE PAIRS TO 10⁸

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1. PRELIMINARIES

A reduced amicable pair is a pair of natural numbers, m and n, such that

$$m = \sigma(n) - n - 1; \quad n = \sigma(m) - m - 1,$$

where σ is the sum of divisors function. Jerrard and Temperley [4] studied numbers k satisfying $k = \sigma(k) - k \pm 1$ which they named *almost perfect numbers*. Lal and Forbes [5] first studied reduced amicable pairs and discovered nine pairs with smaller number $\leq 10^5$. (They coined the name "reduced amicable pair.") In an earlier paper [1], we extended the search to pairs with smaller number $\leq 10^6$, finding six new pairs. Hagis and Lord [3] extended the list to 10^7 , discovering thirty-one new pairs, including two missed in [1]. The present paper extends the listing to 10^8 . The paper [1] included a study of pairs m and n satisfying

$$m = \sigma(n) - n + 1; \quad n = \sigma(m) - m + 1,$$

called *augmented amicable pairs* and listed all pairs with smaller number less than 10^6 . There were nine plus two other pairs both of whose elements exceeded one million. These arose from iterating the function $s_+(n) = \sigma(n) - n + 1$ on integers less than one million. A computer search extended the list to all pairs with smaller number less than one hundred million. Table 2 lists the pairs with one element less than ten million, except for powers of 2. Powers of 2 are fixed points of s_+ and are not included here. A complete list of the 84 pairs up to 10^8 is available from either author. The searches were carried out on the CRAY Y-MP at the University of Illinois at Urbana-Champaign, on Sun 4 work stations at the University of Northern Iowa, and on NeXT and Macintosh IIci stations at California State University, Fresno. Over half the search was run twice, once each at the latter two institutions.

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2. TABLES OF REDUCED AND AUGMENTED AMICABLE PAIRS

The tables of reduced and augmented amicable pairs follow.

TABLE 1. Reduced Amicable Pairs from 10⁷ to 10⁸

1.	12146750 = 5(3).7.11.631;	16247745 = 3(2).5.127.2843
2.	12500865 = 3(3).5.13.17.419;	12900734 = 1.7.11.19.4409
3.	13922100 = 2(2).3(2).5(2).31.499;	31213899 = 3(2).1549.2239
4.	14371104 = 2(5).3.11.31.439;	28206815 = 5.7.13.47.1319
5.	22013334 = 2.3(2).7.17.43.239;	37291625 = 5(3).7.17.23.109
6.	22559060 = 2(2).5.47.103.233;	26502315 = 3.5.7.83.3041
7.	23379224 = 2(3).11.23.11551;	26525415 = 3.5.7(2).151.239
8.	23939685 = 3(3).5.7(3).11.47;	31356314 = 2.11.23.31.1999
9.	26409320 = 2(3).5.7.257.367;	41950359 = 3(3).11.127.1031
10.	27735704 = 2(3).17.109.1871;	27862695 = 3(2).5.7.197.449
11.	28219664 = 2(4).11.109.1471;	32014575 = 3(3).5(2).43.1103
12.	33299000 = 2(3).5(3).7.67.71;	58354119 = 3(2).29.47.67.71
13.	34093304 = 2(3).97.31.41.479;	43321095 = 3(3).5.223.1439
14.	37324584 = 2(3).3(3).11.23.683;	80870615 = 5.7.17.199.683
15.	40818855 = 3.5.7.11.59.599;	42125144 = 2(3).23.179.1279
16.	41137620 = 2(2).3.5.17.31.1301;	84854315 = 5.7.13.251.743
17.	49217084 = 2(2).7.47.149.251;	52389315 = 3(3).5.11.35279
18.	52026920 = 2(3).5.11.23.53.97;	85141719 = 3(3).13.107.2267
19.	52601360 = 2(4).5.7.29.41.79;	97389039 = 3.11.17.173599
20.	61423340 = 2(2).5.11.23.61.199;	88567059 = 2.7(3).17.61.83
21.	62252000 = 2(5).5(3).79.197;	93423519 = 3(2).7.107.13859
22.	64045904 = 2(4).13.367.839;	70112175 = 3.5(2).7.83.1609
23.	66086504 = 2(3).11.750983;	69090615 = 3(2).5.11.29.4813
24.	66275384 = 2(3).7.17.43.1619;	87689415 = 3.5.11.179.2969
25.	68337324 = 2(2).3(3).11.23.41.61;	141649235 = 5.7.13.419.743
26.	72917000 = 2(3).5(3).13.71.79;	115780599 = 3(2).11.47.149.167
27.	76011992 = 2(3).7.179.7583;	87802407 = 3(3).7.11.157.269
28.	77723360 = 2(5).5.511.13.43.79;	145810719 = 3(3).41.107.1231
29.	89446860 = 2(2).3(2).5.17.29231;	197845235 = 5.7.17.332513
30.	93993830 = 2.5.7.727.1847;	99735705 = 3(5).5.23.43.83
31.	94713300 = 2(2).3(4).5(2).11.1063;	240536075 = 5(2).13.37.83.241
32.	94970204 = 2(2).7.107.31699;	96751395 = 3(3).5.13.29.1901
33.	97797104 = 2(4).19.23.71.197;	114332175 = 3(3).5(2).107.1583

Conjecture 0: There are infinitely many reduced (augmented) amicable pairs.

All pairs found have opposite parity. Since $\sigma(n) = m + n \pm 1 = \sigma(m)$, *m* and *n* have the same parity iff $\sigma(n) = \sigma(m)$ are odd iff odd prime factors in *m* and *n* occur only in even powers. Thus, we have

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Conjecture 1: The numbers in a reduced (augmented) amicable pair are of opposite parity.

For each pair, consider the ratio k of the larger number divided by the smaller. In Table 1 the ratios range from 1.0045786 to 2.53962; in Table 2 from 1.0011028 to 2.64749. Thus,

Conjecture 2: For any $\beta > 0$, no matter how small, there exists a reduced (augmented) amicable pair such that $1 < k < 1 + \beta$.

6160 = 2(4).5.7.11;	11697 = 3.7.557
12220 = 2(2).5.13.47;	16005 = 3.5.11.97
23500 = 2(2).5(3).47;	28917 = 3(5).7.17
68908 = 2(2).7.23.107;	76245 = 3.5.13.17.23
249424 = 2(4).7.17.131;	339825 = 3.5(2).23.197
425500 = 2(2).5(3).23.37;	570405 = 2.5.11.3457
434784 = 2(5).3.7.647;	871585 = 5.11.13.23.53
649990 = 2.5.11.19.311;	697851 = 3(2).7.11.19.53
660825 = 3(3).5(2).11.89;	678376 = 2(3).19.4463
1017856 = 2(11).7.71;	1340865 = 3(2).5.83.359
1077336 = 2(3).3(2).13.1151;	2067625 = 5(3).7.17.139
1238380 = 2(2).5.11.13.433;	1823925 = 3.5(2).83.293
1252216 = 2(3).7.59.379;	1483785 = 3(3).5.29.379
1568260 = 2(2).5.19.4127;	1899261 = 3(3).7.13.773
1754536 = 2(3).7.17.19.97;	2479065 = 3.5.29.41.139
2166136 = 2(3).7.47.823;	2580105 = 3.5.11.19.823
2362360 = 2(3).5.7.11.13.59;	4895241 = 3.13.31.4049
2482536 = 2(3).3.7(2).2111;	4740505 = 5.7(2).11.1759
2537220 = 2(2).3.5.7(2).863;	5736445 = 5.11.13.71.113
2876445 = 3(3).5.11.13.149;	3171556 = 2(2).19.29.1439
3957525 = 3(3).5(2).11.13.41;	4791916 = 2(2).41.61.479
4177524 = 2(2).3.13.61.439;	6516237 = 3.7.13.23869
4287825 = 3(2).5(2).17.19.59;	4416976 = 2(4).59.4679
5224660 = 2(2).5.7.67.557;	7524525 = 3.5(2).41.2447
5559510 = 2.3.5.11.17.991;	9868075 = 5(2).7.17.31.107
5641552 = 2(4).7.17.2963;	7589745 = 3(2).5.227.743
5654320 = 2(4).5.7.23.439;	10058961 = 3.11.19.61.263
5917780 = 2(2).5.11.37.727;	8024877 = 3(2).7(2).31.587
6224890 = 2.5.7.17.5231;	7336455 = 3.5.7.107.653
6274180 = 2(2).5.11.19(2).79;	9087741 = 3(3).13.17.1523
6711940 = 2(2).5.17.19.1039;	9012861 = 3(2).11.13.47.149
7475325 = 3.5(2).11.13.17.41;	8273668 = 2(2).13.107.1487
7626136 = 2(3).7.43.3167;	9100905 = 3.5.11.19.2903
7851256 = 2(3).7.19.47.157;	10350345 = 3.5.19.23.1579
7920136 = 2(3).7.233.607;	9152505 = 3(2).5.23.37.239
9026235 = 3(5).5.17.19.23;	9843526 = 2.7.11.41.1559
	6160 = 2(4).5.7.11; 12220 = 2(2).5.13.47; 23500 = 2(2).5(3).47; 68908 = 2(2).7.23.107; 249424 = 2(4).7.17.131; 425500 = 2(2).5(3).23.37; 434784 = 2(5).3.7.647; 649990 = 2.5.11.19.311; 660825 = 3(3).5(2).11.89; 1017856 = 2(11).7.71; 1077336 = 2(3).3(2).13.1151; 1238380 = 2(2).5.11.13.433; 1252216 = 2(3).7.59.379; 1568260 = 2(2).5.19.4127; 1754536 = 2(3).7.17.19.97; 2166136 = 2(3).7.47.823; 2362360 = 2(2).5.7.11.13.59; 2482536 = 2(3).5.7(2).863; 2876445 = 3(3).5.11.13.149; 3957525 = 3(3).5(2).11.13.41; 4177524 = 2(2).3.13.61.439; 4287825 = 3(2).5(2).17.19.59; 5224660 = 2(2).5.7.67.557; 5559510 = 2.3.5.11.17.991; 5641552 = 2(4).7.17.2963; 5654320 = 2(4).5.7.23.439; 5917780 = 2(2).5.11.31.741; 7626136 = 2(3).7.43.3167; 7851256 = 2(3).7.19.47.157; 7920136 = 2(3).7.23.607; 9026235 = 3(5).5.17.19.23;

TABLE 2. Augmented Amicable Pairs to 10⁷

3. THE UNITARY CASE

In [2] searches for the unitary analogues of reduced and augmented amicable pairs to 10^5 were reported. Except for trivial cases, none were found. The search has been extended to 10^6 with no new results.

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