$$\begin{aligned}\sum_{a+b+c+d=n} U_aU_bU_cU_d &= \frac{U_1^3}{6(b^2+4a)^3}[((b^5+7b^3a+12ba^2)n^3-(6b^5+30b^3a+24ba^2)n^2\\ &+(11b^5+17b^3a-48ba^2)n-(6b^5-30b^3a-36ba^2))\,U_{n-2}\\ &+((b^4a+6b^2a^2+8a^3)n^3-(6b^4a+24b^2a^2)n^2\\ &+(11b^4a+6b^2a^2-32a^3)n-(6b^4a-36a^2b^2))\,U_{n-3}].\end{aligned} \tag{11}$$

Proposition 2 now follows from (7), (10), and (11).

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LETTER TO THE EDITOR

Dear Professor Bergum:

The Fibonacci Quarterly readers will be interested in yet another natural occurrence of the Golden Ratio. This occurrence is described in the current issue of *The College Mathematics Journal* (Vol. 28, No. 3, May 1997). On page 205, Peter Schumer (schumer@middlebury.edu) of Middlebury College in Middlebury VT provides an interesting variant on the classical problem of showing that the rectangle with fixed perimeter and maximum area is a square.

Schumer notes that texts often present this problem as the dilemma of a farmer who has a fixed length of fencing and wants to build the most efficient animal pen for grazing. It is a simple calculus problem. The problem is complicated somewhat when the farmer has a fixed length of fencing and is using one side of a barn for all or part of one side of the animal pen. Schumer provides a neat analysis of the optimum pen shape when the length of fencing is some multiple of the length of the barn side used.

When the length of fencing available is $\sqrt{5}$ times the length of the side of barn used, the optimum pen shape is a golden rectangle. This is a neat result, simply derived, of interest to *FQ* readers, and which I have not seen before.

Best regards,

Harvey J. Hindin
Vice-President, Emerging Technologies Group, Inc.