ON t-CORE PARTITIONS

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1. INTRODUCTION

Consider the partition of the natural number n given by

$$n = n_1 + n_2 + \cdots + n_s, \tag{I}$$

where $s \geq 1$ and $n_1 \geq n_2 \geq \cdots \geq n_s$. The Young diagram of this partition consists of the nodes (i, j), where $1 \leq i \leq s$, and for each fixed i, $1 \leq j \leq n_i$. The rightmost node in row i, namely (i, n_i), is called a *hand*. The lowest node in a given column is called a *foot*. At least one node, namely (s, n_s) is both a hand and a foot.

A hand (i, n_i) and a foot (k, j) may be connected by what is known as a hook as follows. Let an *arm* consist of the nodes (i, m) such that $j \leq m \leq n_i$; let a *leg* consist of the nodes (h, j) such that $i \leq h \leq k$. The hook is the union of all nodes in the arm and leg. The corresponding hook number (or hook length) is the number of nodes in the hook, namely $n_i - j + k - i + 1$.

Let the integer $t \geq 2$. We say that a partition is *t-core* if none of the hook numbers are divisible by t. Note that t-core partitions arise in the representation theory of the symmetric group (see [5]); such partitions have also been used to provide new proofs of some well-known results of Ramanujan (see [1]). Let $c_t(n)$ denote the number of t-core partitions of n. It is well known that

$$c_2(n) = \begin{cases} 1 & \text{if } n = \frac{1}{2}m(m+1), \\ 0 & \text{otherwise.} \end{cases}$$

If $n = \frac{1}{2}m(m+1)$, then the unique 2-core partition of n is given by

$$n = m + (m-1) + (m-2) + \cdots + 2 + 1. \tag{II}$$

Recently, Granville and Ono [2] have shown that if $t \geq 4$, then $c_t(n) > 0$ for all n.

In this note, we completely characterize 3-core partitions. We show that they are linked to the quadratic form $x^2 + 3y^2$. As a result, we obtain an independent derivation of Granville and Ono's formula for $c_3(n)$ (see [2]). Finally, we derive recurrences that permit the evaluation of $c_4(n)$ and $c_5(n)$. Note that whereas the formula for $c_5(n)$ given by Garvan et al. [1] requires the canonical factorization of $n+1$, our method for computing $c_5(n)$ does not. We also tabulate these three functions, as well as some related functions, in the ranges $1 \leq n \leq 100$ and $1 \leq n \leq 50$.

2. PRELIMINARIES

Let the integer $n \geq 0$, let the integer $t \geq 2$, let p denote an odd prime, and let x be a complex variable with $|x| < 1$.

Definition 1: Let $c_t(n)$ denote the number of t-core partitions of n.

Definition 2: Let $b_t(n)$ denote the number of partitions of n such that no part is divisible by t.

Definition 3: Let $(a/p) = \begin{cases} \text{Legendre symbol} & \text{if } p \nmid a, \\ 0 & \text{if } p \mid a. \end{cases}$

Definition 4: Let $E(n) = \frac{1}{2}n(3n-1)$.

Lemma:

(1) $\sum_{n=0}^{\infty} c_t(n)x^n = \prod_{n=1}^{\infty}(1-x^{tn})^t/(1-x^n)$;

(2) $\sum_{n=0}^{\infty} b_t(n)x^n = \prod_{n=1}^{\infty}(1-x^{tn})/(1-x^n)$;

(3) $b_t(n) = p(n) + \sum_{k\geq 1}(-1)^k(p(n-tE(k)) + p(n-tE(-k)))$;

(4) $\prod_{n=1}^{\infty}(1-x^n)^3 = \sum_{k=0}^{\infty}(-1)^k(2k+1)x^{\frac{1}{2}k(k+1)}$;

(5) If $n \equiv 4 \pmod 5$, then $p(n) \equiv 0 \pmod 5$.

Remarks: The identities (1) and (2) are well known (see [1], [2], and [7]). Note that (3) follows from (2), (4) is due to Jacobi, and (5) is due to Ramanujan.

Notation: Suppose that a partition of n has r distinct parts and that the summand n_i occurs k_i times, where $1 \leq i \leq r$. Then we occasionally write

$$n = \prod_{i=1}^{r} n_i^{k_i}.$$

Theorem 1: Conjugate partitions have the same hook numbers.

Proof: If $n \geq 1$, consider the map that sends each partition of n to its conjugate. Thus, hands are interchanged with feet, arms with legs, and hooks with hooks having the same hook numbers.

Theorem 2: $c_t(n) = \begin{cases} p(n) & \text{if } n < t, \\ p(t) - t & \text{if } n = t. \end{cases}$

Proof: We define $c_t(0) = p(0) = 1$. If $1 \leq n \leq t-1$, then each hook in a partition of n has length at most $t-1$, so every partition of n is t-core, so $c_t(n) = p(n)$. Now let $n = t$. Each partition $t = (t-j)1^j$, where $0 \leq j \leq t-1$, has a t-hook and thus is not t-core. On the other hand, if the least part in a partition of t is strictly between 1 and t, then each hook number is at most $t-1$, so the partition is t-core. Therefore, $c_t(t) = p(t) - t$.

3. 3-CORE PARTITIONS

By means of Theorems 3 through 8 below, we characterize all 3-core partitions.

Theorem 3: Each of the following partitions is 3-core:

(a) $n = 2m(2m-2)(2m-4)\cdots(4)(2)$;

(b) $n = (2m-1)(2m-3)(2m-5)\ldots(3)(1)$;

(c) $n = m^2(m-1)^2\ldots 2^2 1^2$;

(d) $n = m(m-1)^2(m-2)^2\ldots 2^2 1^2$.

Proof: Since the partitions in (c) and (d) are the conjugates of those in (a) and (b), it suffices, by virtue of Theorem 1, to prove (a) and (b). We first prove (a) by induction on m. The statement is true by Theorem 2 when $m=1$. Let $n'=(2m+2)(2m)(2m-2)\dots(4)(2)$. If we omit the first row or the first two columns in the Young diagram for n', we obtain the Young diagram for n. Therefore, by the induction hypothesis, it suffices to show that all hooks from the new hand, namely $(1, 2m+2)$, to the feet in the last row, namely $(m+1, 1)$ and $(m+1, 2)$, have hook numbers not divisible by 3. These hook numbers are $3m+2$ and $3m+1$, respectively, so we are done.

We sketch the proof of (b), which is similar. Again, the statement is true for $m=1$ by Theorem 2. Let $n''=(2m+1)(2m-1)\dots(3)(1)$. We need only note that the hook from the new hand, namely $(1, 2m+1)$, to the lowest foot, namely $(m+1, 1)$, has hook number $=3m+1$.

Theorem 4: Let $r\geq 1$ and $m\geq 1$. Then each of the following partitions is 3-core:

(a) $n=(m+2r)(m+2r-2)\dots(m+2)m^2(m-1)^2\dots 2^2 1^2$;

(b) $n=(m+2r-1)(m+2r-3)\dots(m+1)m^2(m-1)^2\dots 2^2 1^2$.

Proof: For (a), look at the corresponding Young diagram. By Theorem 3, any hook that occurs entirely in the first r rows or in the last $2m$ rows has length not divisible by 3. Therefore, it suffices to consider hooks from a hand in the first r rows to a foot in the last $2m$ rows. Such a hand has coordinates $(i, m+2r+2-2i)$, where $1\leq i\leq r$; such a foot has coordinates $(r+2j, m+1-j)$, where $1\leq j\leq m$. The corresponding hook has length $3(r-i+j)+2$, so we are done.

The proof for (b) is similar. A hand from the first r rows has coordinates $(i, m+2r+1-2i)$, where $1\leq i\leq r$. Again, a foot from the last $2m$ rows has coordinates $(r+2j, m+1-j)$, where $1\leq j\leq m$, so the corresponding hook has length $3(r-i+j)+1$.

Theorem 5: Let $n=n_1+n_2+\cdots+n_s$ be a 3-core partition of n, where $s\geq 1$. Then the following must hold:

(a) $n_s\leq 2$.

(b) If $n\geq 3$, then $s\geq 2$.

(c) $n_i-n_{i+1}\leq 2$ for all i such that $1\leq i\leq s-1$.

(d) Each part occurs at most twice.

(e) If $n_{i+1}=n_i$, then either (i) $1\leq i\leq s-2$ and $n_{i+2}=n_{i+1}-1$ or (ii) $i=s-1$ and $n_{s-1}=1$.

(f) If $n_{i+1}=n_i-1$, then $1\leq i\leq s-2$ and $n_{i+2}=n_{i+1}$.

Proof: A partition such that any of (a) through (f) fails to hold has a hook of length 3.

Theorem 6: $c_3(n)$ is the number of distinct ways that n can be represented in the form

$$n=r(r+m+k)+m(m+1),$$

where $k=0$ or 1, $r\geq 0, m\geq 0$, and $rm>0$. For each such representation, the corresponding 3-core partition of n is given by

$$n=(m+2r+k-1)(m+2r+k-3)\dots(m+k+1)m^2(m-1)^2\dots 2^2 1^2.$$

Proof: The conclusion follows from Theorems 4 and 5, and from the hypothesis.

Remark: Note that m is the number of parts that occur twice, while r is the number of parts that occur once.

Theorem 7: n has a self-conjugate 3-core partition iff there exists $r \geq 1$ such that $n = 3(3r \pm 2)$. If such a self-conjugate 3-core partition of r exists, then it is unique.

Proof: If n has a self-conjugate 3-core partition, then the number of parts must equal the largest part. Therefore, by Theorem 6, we must have $r + 2m = m + 2r + k - 1$, with k, m, and r as in the hypothesis of Theorem 6. Thus, $m = r + k - 1$. If $k = 0$, then $n = r(2r-1) + r(r-1) = r(3r-2)$; if $k = 1$, then $n = r(2r+1) + r(r+1) = r(3r+2)$. Conversely, the partitions

$$n = (3r-2)(3r-4)\dots(r+2)(r(r-1)^2(r-2)^2\dots 1^2$$

and

$$n = 3r(3r-2)\dots(r+2)r^2(r-1)^2(r-2)^2\dots 1^2$$

are 3-core by Theorem 4, and are self-conjugate. Uniqueness follows from the fact that n has at most a single representation, $n = r(3r \pm 2)$.

Corollary 1: $c_3(n) \equiv \begin{cases} 1 \pmod 2 & \text{if } n = r(3r \pm 2), \\ 0 \pmod 2 & \text{otherwise.} \end{cases}$

Proof: This follows from Theorems 1 and 7.

Corollary 2: $c_3(n)$ changes parity infinitely often as n tends to infinity.

Proof: This follows from Corollary 1.

Theorem 8: $c_3(n)$ is the number of solutions of the equation $x^2 + 3y^2 = 12n + 4$ such that $x \geq 1$ and $y \geq [n^{1/2}]$ if $n > 0$.

Proof: By Theorem 6, each 3-core partition of n corresponds to a solution of

$$n = r(r + m + k) + m(m+1),$$

where $k = 0$ or 1, $r \geq 0$, $m \geq 0$, and $rm > 0$. Let $v = m + k$, so $v \geq 0$. Then $n = r(r+v) + v(v \pm 1)$, so that $12n + 4 = (3v \pm 2)^2 + 3(v + 2r)^2$. Let $x = 3v + 2(-1)^k$ and $y = v + 2r$. This yields

$$x^2 + 3y^2 = 12n + 4.$$

If $v = 0$, then $m = k = 0$, so $x = 2$. If $v \geq 1$, then $x \geq 3v - 2 \geq 1$. Thus, in all cases, $x \geq 1$. Now suppose that $y < [n^{1/2}]$. Since $y = v + 2r$ and $r \geq 0$, this implies that $v < [n^{1/2}]$, so $v \leq [n^{1/2}] - 1$. Since $y < [n^{1/2}]$, we must have $x > 3n^{1/2}$, that is, $3v \pm 2 > 3n^{1/2}$, hence $v > n^{1/2} - \frac{2}{3}$. This implies that $n^{1/2} - [n^{1/2}] < -\frac{1}{3}$, an impossibility. Thus, $y \geq [n^{1/2}]$. Conversely, suppose that $x^2 + 3y^2 = 12n + 4$, where $x \geq 1$ and $y \geq [n^{1/2}]$. Since $3 \nmid x$, we may let $x = 3v + 2(-1)^k$, where v is an integer and $k = 0$ or 1. Since $x \equiv y \equiv v \pmod 2$, we may let $y = v + 2r$, where r is an integer.

If $k = 0$, then $v = (x-2)/3$, so $v \geq -\frac{1}{3}$. Since v is an integer, we have $v \geq 0$. If $k = 1$, then $v = (x+2)/3$, so $v \geq 1$. Let $m = v - k$. In either case, we have $m \geq 0$. Since $y \geq [n^{1/2}]$, we have $x^2 \leq 12n - 3[n^{1/2}]^2 + 4$, that is, $x^2 \leq 9n + 3(n - [n^{1/2}]^2) + 4$. But $n - [n^{1/2}]^2 \leq 2[n^{1/2}]$, so we have $x^2 \leq 9n + 6[n^{1/2}] + 4$. Hence $x^2 \leq (3n^{1/2} + 1)^2 + 3$, so that $x \leq ((3n^{1/2} + 1)^2 + 3)^{1/2}$, which implies $x \leq 3n^{1/2} + 1$.

If $k = 0$, we have $3v + 2 \le 3n^{1/2} + 1$, hence $v \le n^{1/2} - \frac{1}{3}$. Now $r = \frac{1}{2}(y - v)$, so

$$r \ge \tfrac{1}{2}([n^{1/2}] - n^{1/2} + \tfrac{1}{3}) > \tfrac{1}{2}(-\tfrac{2}{3}) = -\tfrac{1}{3}.$$

If $k = 1$, we have $3v - 2 \le 3n^{1/2} + 1$, hence $v \le n^{1/2} + 1$, $v \le [n^{1/2}] + 1$. Thus,

$$r \ge \tfrac{1}{2}([n^{1/2}] - [n^{1/2}] - 1),$$

that is, $r \ge -\frac{1}{2}$. In either case, since r is an integer, we must have $r \ge 0$.

Finally, if we let $x = 3v \pm 2$ and $y = v + 2r$, and substitute into $x^2 + 3y^2 = 12n + 4$, then, after simplifying, we obtain

$$n = v(v \pm 1) + r(r + v).$$

If $k = 0$, then $v = m$, so

$$n = m(m+1) + r(r+m).$$

If $k = 1$, then $v = m + 1$, so

$$n = m(m+1) + r(r+m+1).$$

Thus, we have

$$n = m(m+1) + r(r+m+k).$$

Since $n > 0$, we must have $rm > 0$.

Lemma 1: Consider the equation

$$x^2 + 3y^2 = 12n + 4. \qquad (*)$$

The number of solutions of $(*)$ such that $|y| \ge [n^{1/2}]$ is $4\sigma(3n+1)$, where $\sigma(n) = \sum\{(d/3) : d \mid n\}$. (Here we are following the notation of [2].)

Proof: Let $12n + 4 = 2^k m$, where $k \ge 2$ and $2 \nmid m$. According to [4] (p. 308, Ex. 3), if j is the number of solutions of $(*)$, then $j = 6\sigma(3n+1)$. We must show that if j' is the number of solutions of $(*)$ such that $|y| \ge [n^{1/2}]$, then $j' = 4\sigma(3n+1)$.

Suppose that $x = a$, $y = b$ is a solution of $(*)$. Let $\omega = \exp(2\pi i/3)$. Passing to $Q(\omega)$, we have

$$(a + b\sqrt{-3})(a - b\sqrt{-3}) = 12n + 4.$$

Let $z_1 = (a+b) + 2b\omega = a + b\sqrt{-3}$. Then $N(z_1) = a^2 + 3b^2 = 12n + 4$. However, $Q(\omega)$ has 6 units, namely, $\pm 1, \pm\omega, \pm\omega^2$, so we obtain additional solutions of $(*)$ corresponding to

$$z_2 = \omega z_1,\ z_3 = \omega^2 z_1,\ z_4 = -z_1,\ z_5 = -z_2,\ z_6 = -z_3.$$

Now $z_2 = -2b + (a-b)\omega$ and $z_3 = (b-a) - (a+b)\omega$, so it suffices to show that if $|y| < [n^{1/2}]$, then $|x \pm y| \ge 2[n^{1/2}]$. By hypothesis, we have $|x|^2 + 3|y|^2 = 12n + 4$, so

$$|x|^2 = 12n + 4 - 3|y|^2 > 12n - 3[n^{1/2}]^2 + 4 \ge 9n + 4.$$

Thus $|x| > 3n^{1/2}$. Now

$$|x \pm y| \ge |x| - |y| \ge 3n^{1/2} - [n^{1/2}] \ge 2[n^{1/2}],$$

so we are done.

Theorem 9: $c_3(n) = \sigma(3n+1) = \Sigma\{(d/3) : d \mid (3n+1)\}$.

Proof: This follows from Theorem 8 and Lemma 1, omitting solutions of (8) such that $x < 0$ or $y < 0$.

Remark: An alternate proof of Theorem 9, based on the theory of modular forms, was given in [2].

Theorem 10: If there exists $k \geq 1$ such that $3n \equiv 2^{2k-1} - 1 \pmod{2^{2k}}$, then $c_3(n) = 0$.

Proof: By Theorem 8 and [4] (p. 308, Ex. 3), we have $c_3(n) = 0$ if $12n + 4 = 2^{2k+1}m$, where $k \geq 1$ and $2 \nmid m$. That is, $c_3(n) = 0$ if $3n \equiv 2^{2k-1} - 1 \pmod{2^{2k}}$ for some $k \geq 1$.

Corollary 3: $c_3(n) = 0$ if $n \equiv 3 \pmod 4$, $n \equiv 13 \pmod{16}$, $n \equiv 53 \pmod{64}$, etc.

Proof: This follows from Theorem 10.

Theorem 11: $c_3(n)$ is unbounded as n tends to infinity.

Proof: Let $n = (7^{k-1} - 1)/3$. Then $c_3(n) = \sigma(7^{k-1}) = k$. Since k is arbitrary, we are done.

Table 1 below lists $c_3(n)$ for all n such that $1 \leq n \leq 100$.

TABLE 1

n	$c_3(n)$	n	$c_3(n)$	n	$c_3(n)$	n	$c_3(n)$
1	1	26	2	51	0	76	2
2	2	27	0	52	2	77	0
3	0	28	0	53	0	78	0
4	2	29	0	54	0	79	0
5	1	30	4	55	0	80	2
6	2	31	0	56	3	81	2
7	0	32	2	57	2	82	4
8	1	33	1	58	2	83	0
9	2	34	2	59	0	84	0
10	2	35	0	60	2	85	1
11	0	36	2	61	0	86	4
12	2	37	2	62	0	87	0
13	0	38	0	63	0	88	0
14	2	39	0	64	2	89	2
15	0	40	1	65	3	90	2
16	3	41	2	66	2	91	0
17	2	42	2	67	0	92	2
18	0	43	0	68	0	93	0
19	0	44	4	69	2	94	2
20	2	45	0	70	2	95	0
21	1	46	2	71	0	96	1
22	2	47	0	72	4	97	2
23	0	48	0	73	0	98	0
24	2	49	2	74	2	99	0
25	2	50	2	75	0	100	4

4. 4-CORE PARTITIONS

This subject has recently been explored in some detail (see [3] and [8]). The following theorem permits the evaluation of $c_4(n)$.

Theorem 12: $c_4(n)=\sum_{k=0}^{\infty}(1)^k(2k+1)b_4(n-2k(k+1))$.

Proof: Equation (1) implies

$$\begin{aligned}\sum_{n=0}^{\infty}c_4(n)x^n&=\prod_{n=1}^{\infty}(1-x^{4n})^4/(1-x^n)\\&=\prod_{n=1}^{\infty}(1-x^{4n})/(1-x^n)\prod_{n=1}^{\infty}(1-x^{4n})^3\\&=\left(\sum_{n=0}^{\infty}b_4(n)x^n\right)\left(\prod_{n=1}^{\infty}(1-x^{4n})^3\right)\end{aligned}$$

by (2). Let

$$g_4(n)=\begin{cases}(-1)^m(2m+1) & \text{if } n=2m(m+1),\\ 0 & \text{otherwise.}\end{cases}$$

Then (4) implies

$$\begin{aligned}\sum_{n=0}^{\infty}c_4(n)x^n&=\left(\sum_{n=0}^{\infty}b_4(n)x^n\right)\left(\sum_{n=0}^{\infty}g_4(n)x^n\right)\\&=\sum_{n=0}^{\infty}\left(\sum_{k=0}^{\infty}b_4(n-k)g_4(k)\right)x^n.\end{aligned}$$

Matching coefficients of like powers of x, we get

$$c_4(n)=\sum_{k=0}^{\infty}b_4(n-k)g_4(k),$$

from which the conclusion follows.

5. 5-CORE PARTITIONS

Garvan, Kim, and Stanton [1] have shown that

$$c_5(n)=\sum_{d\,|\,(n+1)}(d/5)\frac{n+1}{d}.$$

In order to use this formula, one needs to know the divisors (or, equivalently, the canonical factorization) of $n+1$. We now present an alternative method of computing $c_5(n)$ that does not require factorization.

Theorem 13: Let

$$f_5(n)=b_5(n)+\sum_{k\geq 1}(-1)^k(b_5(n-5E(k))+b_5(n-5E(-k))).$$

Then

$$c_5(n)=\sum_{j=0}^{\infty}(-1)^j(2j+1)f_5(n-5j(j+1)/2).$$

Proof: Equation (1) implies

$$\begin{aligned}\sum_{n=0}^{\infty}c_5(n)x^n &= \prod_{n=1}^{\infty}(1-x^{5n})^5/(1-x^n)\\ &= \prod_{n=1}^{\infty}(1-x^{5n})^2/(1-x^n)\prod_{n=1}^{\infty}(1-x^{5n})^3.\end{aligned}$$

Now

$$\begin{aligned}\prod_{n=1}^{\infty}(1-x^{5n})^2/(1-x^n) &= \prod_{n=1}^{\infty}(1-x^{5n})/(1-x^n)\prod_{n=1}^{\infty}(1-x^{5n})\\ &= \left(\sum_{n=0}^{\infty}b_5(n)x^n\right)\prod_{n=1}^{\infty}(1-x^{5n})\\ &= \sum_{n=0}^{\infty}f_5(n)x^n\end{aligned}$$

by (2) and the definition of $f_5(n)$. Also, by (4), we have

$$\prod_{n=1}^{\infty}(1-x^{5n})^3=\sum_{n=0}^{\infty}g_5(n)x^n,$$

where

$$g_5(n)=\begin{cases}(-1)^k(2k+1) & \text{if } n=5k(k+1)/2,\\ 0 & \text{otherwise.}\end{cases}$$

Thus, we have

$$\begin{aligned}\sum_{n=0}^{\infty}c_5(n)x^n &= \left(\sum_{n=0}^{\infty}f_5(n)x^n\right)\left(\sum_{n=0}^{\infty}g_5(n)x^n\right)\\ &= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}f_5(n-k)g_5(k)\right).\end{aligned}$$

Matching coefficients of like powers of x, we obtain

$$c_5(n)=\sum_{k=0}^{n}f_5(n-k)g_5(k),$$

from which the conclusion follows.

Table 2 below lists $b_4(n)$, $c_4(n)$, $b_5(n)$, $f_5(n)$, and $c_5(n)$ for each n such that $1\le n\le 50$.

Our final theorem is inspired by examination of Table 2.

Theorem 16: If $n\equiv 4 \pmod 5$, then $b_5(n)\equiv f_5(n)\equiv c_5(n)\equiv 0 \pmod 5$.

Proof: By virtue of Theorem 15 and the definition of $f_5(n)$, it suffices to show that $b_5(n)\equiv 0 \pmod 5$ when $n\equiv 4 \pmod 5$. This follows from (3) and (5).

TABLE 2

n	$b_4(n)$	$c_4(n)$	$b_5(n)$	$f_5(n)$	$c_5(n)$
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	1	5	5	5
5	6	3	6	5	2
6	9	3	10	9	6
7	12	3	13	11	5
8	16	4	19	16	7
9	22	4	25	20	5
10	29	2	34	27	12
11	38	2	44	33	6
12	50	7	60	45	12
13	64	3	76	54	6
14	82	5	100	70	10
15	105	6	127	87	11
16	132	2	164	110	16
17	166	4	205	132	7
18	208	7	262	167	20
19	258	3	325	200	15
20	320	4	409	248	12
21	395	7	505	297	12
22	484	5	628	363	22
23	592	8	769	431	10
24	722	5	950	525	25
25	876	4	1156	621	12
26	1060	4	1414	746	20
27	1280	8	1713	882	18
28	1539	5	2081	1053	30
29	1846	6	2505	1235	10
30	2210	7	3026	1467	32
31	2636	2	3625	1716	21
32	3138	9	4352	2024	24
33	3728	11	5192	2361	16
34	4416	3	6200	2770	30
35	5222	8	7364	3217	21
36	6163	9	8756	3762	36
37	7256	4	10357	4354	20
38	8528	6	12258	5064	24
39	10006	5	14450	5850	25
40	11716	7	17034	6777	42
41	13696	5	20006	7799	12
42	15986	14	23500	9009	42
43	18624	7	27510	10341	36
44	21666	4	32200	11900	35
45	25169	10	37582	13627	22
46	29190	5	43846	15633	46
47	33808	10	51022	17583	22
48	39104	11	59353	20430	43
49	45164	3	68875	23275	25
50	52098	9	79888	26555	32

REFERENCES

1. Frank Garvan, Dongsu Kim, & Dennis Stanton. "Cranks and t-Cores." *Inventiones Math.* **101** (1990):1-17.
2. Andrew Granville & Ken Ono. "Defect Zero p-Groups for Finite Simple Groups." *Trans. Amer. Math. Soc.* **148** (1996):331-47.
3. Michael D. Hirschmann & James A. Sellers. "Some Amazing Facts about 4-Cores." *J. Number Theory* **60** (1996):51-69.
4. L. K. Hua. *Introduction to Number Theory*. New York: Springer-Verlag, 1982.
5. Gordon James & Adalbert Kerber. *The Representation Theory of the Symmetric Group*. New York: Addison-Wesley, 1981.
6. Edmund Landau. "Ueber die Einteilung der positiven ganzen Zahlen in view Klassen nach der Mindestzahl der zu ihr arritiven 2-Sammensetzung erforderlichen Quadrate." *Archiv. der Math. u. Phy.* **13.3** (1908):305-12.
7. Ken Ono, Neville Robbins, & Brad Wilson. "Some Recurrences for Arithmetical Functions." *J. Indian Math. Soc.* **62.1** (1996):1-22.
8. Ken Ono & Lawrence Sze. "4-Core Partitions and Class Numbers." *Acta Arith.* **80.3** (1997):249-72.

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