STAR GEOMETRY

Pythagoras, Fibonacci and Beard

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The diagonal of the pentagon is made the unit of length in the upper left circle of the accompanying drawings. K is the ratio of the side of the pentagon to its diagonal.

Line 4.3 is the short side of triangle 1-4-3 and the long side of triangle 3-4-6.

Line 4.6 is the short side of triangle 3-4-6 and the long side of triangle 4-6-7.

Therefore the sides of these three similar isoseeles triangles are respectively 1 and K, K and K^2, K^2 and K^3. Lines 1-6 and 3-6 have the same length, K, as they are the equal sides of the isosceles triangle 1-3-6. Diagonal 1-4 of unit length is thus divided into segments K and K^2, that is $K^2 + K = 1$.

It follows from this equation that

$$K = \frac{1}{2}\sqrt{5} - \frac{1}{2} = 0.618034$$

and that $K^n = K^{n+1} + K^{n+2}$.

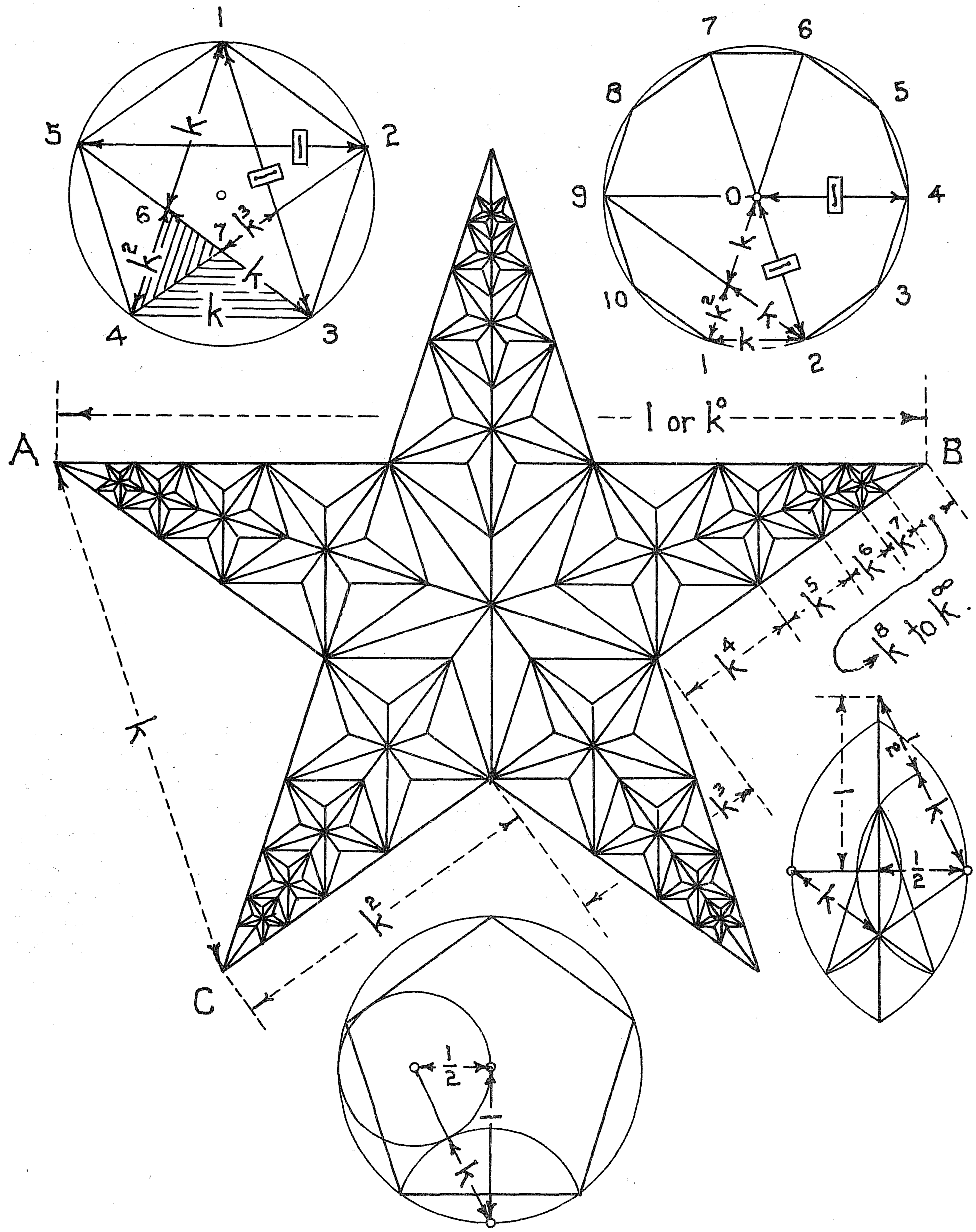
Since each power of K is the sum of its next two higher powers, these powers form a Fibonacci series when arranged in their descending order.

The radius is made the unit of length in the upper right circle. Similar triangles divide radius O-1 into segments K and K^2. This construction demonstrates that each side of a regular decagon has the golden section ratio to the radius of its circumscribing circle.

The right triangle in the lower circle has sides of 1, 1/2 and $1/2\sqrt{5}$. The dimension K with the value of $1/2\sqrt{5} - 1/2$ is the length of the side of the inscribed decagon.

The leaf shaped figure shows one simple way to construct a five pointed star of a given width.

The Fibonacci star of stars is proportioned in the ten successive powers of the golden section from K^0 to K^9.



1
2
3
4
5
6
7
8
9
10
0
k
k^2
k^3
1 or k^0
A
B
C
k^4
k^5
k^6
k^7
k^8 to k^∞.
$\frac{1}{2}$

AB is made the unit of length or K^0.

AC, the side of the bounding pentagon has the length of K.

The side of each ray of the master star is K^2 long.

The bounding pentagon of the central star has sides of K^3 length.

Since one diagonal of each of the smaller stars is a side of the bounding pentagon of the next larger star all of the corresponding dimensions of these successive stars are in golden section ratio.

The bounding pentagons of the successive stars have sides of K^4, K^5, K^6 and K^7 respectively.

The rays of the smallest stars have K^8 edges and base widths of K^9.

This figure can be used to demonstrate that any power of the golden section K^n, is the sum of all of its higher powers from K^{n+2} to K^∞.

The lines connecting the tips of the central star and the centers of the five next smaller stars form a decagon.

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