ADVANCED PROBLEMS AND SOLUTIONS

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Send all communications concerning Advanced Problems and Solutions to Verner E. Hoggatt, Jr., Mathematics Department, San Jose State College, San Jose, California. This department especially welcomes problems believed to be new or extending old results. Proposers should submit solutions or other information that will assist the editor. To facilitate their consideration, solutions should be submitted on separate signed sheets within two months after publication of the problems.

H-84 *Proposed by Oswald Wyler, Carnegie Institute of Technology*

For any positive integer m, there is an integer $k(m)$ with the property that m divides a Fibonacci number F_n if and only if $k(m)$ divides n. For a prime number p, it is known that $k(p)$ divides $p - 1$ if $p \equiv \pm 1 \pmod 5$, and that $k(p)$ divides $p + 1$ if $p \equiv \pm 2 \pmod 5$. Using a table of Fibonacci numbers, one finds that $F_{k(p)}$ is a multiple of p, but not of p^2, for small prime numbers p, or in other words that $k(p) < k(p^2)$ for small prime numbers p. It does not seem to be known whether this is the case for all prime numbers p or not.

H-85 *Proposed by H.W. Gould, West Virginia University, Morgantown, West Va.*

Let

$$D_n = f_n x^n - [f_n x^n] ,$$

where

$$F_{n+1} = f_n + f_{n-1} \text{ with } f_0 = f_1 = 1, \quad x = (1 + \sqrt{5})/2 ,$$

and $[z]$ = greatest integer $\leq z$ (so that $z - [z]$ = fractional part of z). Prove (or disprove) the existence of the limits

$$\lim_{n \to \infty} D_{2n} = 0.27\ldots = A$$

and

$$\lim_{n \to \infty} D_{2n+1} = 0.72\ldots = B$$

with $A + B = 1$. Generalize to case of $u_{n+1} = pu_n + qu_{n-1}$, where p and q are real and u_0 and u_1 are given.

H-86 *Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, Calif.*

Let p and q be integers such that $p + q \geq 1$, $q \geq 0$ show that if

$$x^p(x-1)^q - 1 = 0 \text{ has roots } r_1, r_2, \ldots, r_{p+q}$$

and

$$(x-1)^{p+q} - x^p = 0 \text{ has roots } s_1, s_2, \ldots, s_{p+q}$$

then

$$s_i^{p+q} = r_i^q \text{ for } i = 1, 2, 3, \ldots, p+q \quad .$$

H-87 *Proposed by Monte Boisen, Jr., San Jose State College, San Jose, Calif.*

Show that, if

$$u_0 = u_2 = u_3 = \ldots = u_{n-1} = 1 \quad \text{and}$$

$$u_k = u_{k-1} + u_{k-2} + \ldots + u_{k-n} \quad k \geq n \quad ,$$

then

$$\frac{1 - x^2 - 2x^3 - \ldots - (n-2)x^{n-1}}{1 - x - x^2 - \ldots - x^n} = \sum_{k=0}^{\infty} u_k x^k \quad .$$

H-88 *Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, Calif.*

$$\sum_{k=0}^{n} F_{4mk} \binom{n}{k} = L_{2k}^m F_{2km} \quad ,$$

where F_m and L_m are the mth Fibonacci and mth Lucas numbers, respectively.

H-80 *Proposed by J.A.H. Hunter, Toronto, Ontario, Canada and Max Rumney, London, England – Corrected*

Show

$$\sum_{r=0}^{n} \binom{n}{r} F_{r+2}^2 = \sum_{r=0}^{n} \binom{n-1}{r} F_{2r+5} \quad .$$

SOLUTIONS

THE FINAL WORD

H-42 The corrected list is:

1, 2, 3, 5, 9, 15, 20, 25, 41 .

J. D. Konhauser first noted the typing error.

Solution to the Crossword Puzzle by H.W. Gould, West Virginia University

B		F	I	B	O	N	A	C	C	I			T
S	E	R	I	E			L		O	S	A	G	E
		A		L	C	M		U	N		T	E	N
P	H	Y	L	L	O	T	A	X	I	S		N	
A			E		E		D	I	C	E		E	P
S	I	N	O		F		D		A	Q		R	I
C	A	I	N		F	E			L	U	C	A	S
A	L	M	A		I	Q		U		E		T	A
L			R	E	C	U	R	R	E	N	C	E	
	H	A	D	R	I	A	N			C	O	S	H
T	Y	R	O		E	T		U	S	E	S		I
A		A			N	E	X	T			E	R	S
R	A	B	B	I	T	S		E	V	I	C	T	S

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