

(K)

$$\text{The circulant } \begin{vmatrix} 1 & 9 & 6 & 7 \\ 7 & 1 & 9 & 6 \\ 6 & 7 & 1 & 9 \\ 9 & 6 & 7 & 1 \end{vmatrix} = -3^2(23)(29) .$$

$$\begin{vmatrix} 1 & 7 \\ 6 & 9 \end{vmatrix} \text{ divides } \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 6 & 7 & 6 \\ 6 & 7 & 6 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix}, \text{ that is, } \frac{3(11)^2}{-3(11)} = -11 .$$

$$\begin{vmatrix} 1 & 9 \\ 7 & 6 \end{vmatrix} \text{ divides } \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 1 & 1 & 6 \\ 6 & 1 & 1 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix}, \text{ that is, } \frac{9(11)(19)}{-3(19)} = -33 .$$

$$\begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 9 & 9 & 6 \\ 6 & 9 & 9 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 9^3 \cdot \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 6 & 6 & 6 \\ 6 & 6 & 6 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 3^3(43) \cdot \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 7 & 7 & 6 \\ 6 & 7 & 7 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 3^2(113) .$$

$$\begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 0 & 0 & 6 \\ 6 & 0 & 0 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 45^2 \cdot \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 6 & 7 & 0 \\ 6 & 7 & 0 & 0 \\ 7 & 0 & 0 & 0 \end{vmatrix} = 7^4 .$$

(Continued from p. 473.)

Of the twenty-four four-digit numbers that can be written with the digits of 1967, seven are prime:

1697, 6197, 6719, 6791, 6917, 6971, and 7691 .

(F)

$$- \begin{vmatrix} 1 & 9 \\ 6 & 7 \end{vmatrix} = 47 .$$
