A DIGITAL BRACELET FOR 1967

CHARLES W. TRIGG
San Diego, California

A bracelet is one period of a simply periodic series considered as a closed sequence with terms equally spaced around a circle. Thus distances between terms may be measured in degrees. A bracelet may be regenerated by starting at any arbitrary point to apply the generating law. A bracelet may be cut at any arbitrary point for straight line representation without loss of any properties.

A digital bracelet may be constructed by starting with a sequence of four digits, affixing the units' digit of their sum, again affixing the units' digit of the sum of the last four digits, and continuing the process.

Starting with 1967 this process will generate the sequence

1 9 6 7 3 5 1 6 5 7 9 7 8 1 5 ···

in which four odd digits and one even digit alternate throughout. Since there are only 5^4 sets of four ordered odd digits, the sequence must repeat in not over $5(5^4)$ or 3125 operations. In fact, it does repeat after 1560 operations producing a bracelet of 1560 digits. The complete bracelet is given on page 480.

This bracelet could be said to belong to 1967, but 1560 years have an equal claim to it, for example, the following from the twentieth century:

1901	1923	1935	1949	1957	1978	1991	1999
1903	1929	1937	1951	1958	1979	1992	
1907	1930	1938	1952	1967	1983	1994	
1912	1932	1941	1953	1973	1985	1996	
1917	1933	1947	1956	1974	1987	1997	

By retaining only the units' digits in the generation of the series we actually reduced each sum modulo 10. To be consistent we will reduce modulo 10 the results of all operations (such as multiplication) to which the elements of the bracelet are subjected. Thus we deal only with digits in a modular arithmetic wherein 3, 9, 7, 1 is a cyclic geometric progression.

In order to establish relationships between equidistant digits the bracelet may be written in several rows of various but equal lengths so that each digit column consists of equidistant digits.

Digits 180^0 apart may be written in two rows:

19673 51657 97815 15231 17211 15859 79051 51297 97253 77295 39631 ···
91437 59453 13295 95879 93899 95251 31059 59813 13857 33815 71479 ···

So each pair of diametrically opposite digits sum to zero, and the sum of all the digits in the bracelet is zero.

All the digits 120^0 apart in the bracelet may be exhibited in three rows:

19673 51657 97815 15231 17211 15859 79051 51297 97253 77295 39631 ···
97411 39473 37033 39833 37695 77879 15275 93417 57091 77497 77015 ···
59071 75035 31217 11091 11259 73437 71839 11451 11811 11473 59419 ···

Each column of pentads is composed of two odd-digit, one even-digit, and two odd-digit columns. Each pentad column sums to 55055. The digit columns encompass all the sets of three odd integers that sum to 5 except 5, 5, 5 and all the sets of three even integers, other than 0, 0, 0, which sum to zero.

When the digits of the bracelet are written in four equal rows the digits in each column are 90^0 apart. Thus

19673 51657 97815 15231 17211 15859 79051 51297 97253 77295 39631 ···
37819 53851 71435 35693 31633 35457 17053 53671 71659 11675 97893 ···
91437 59453 13295 95879 93899 95251 31059 59813 13857 33815 71479 ···
73291 57259 39675 75417 79477 75653 93057 57439 39451 99435 13217 ···

Each column of digits is a cyclic permutation of 3, 9, 7, 1; 6, 8, 4, 2; 5, 5, 5, 5; or 0, 0, 0, 0. Hence each column is in geometric progression with $r = 3$. So successive multiplication by 3 will rotate the bracelet counterclockwise in 90^0 jumps. The same result is obtained by multiplying the bracelet by 3, 9, 7, 1 in order. The sums of the pentads form the array

6	4	0	2	2	8	2	4	6	0	2
8	2	0	6	6	4	6	2	8	0	6
4	6	0	8	8	2	8	6	4	0	8
2	8	0	4	4	6	4	8	2	0	4

Each of these columns is in G. P. with $r = 3$.

When the digits of the bracelet are written in five equal rows the digits in each column are 72^0 apart. Thus

19673	51657	97815	15231	17211	15859	79051	51297	97253	77295	39631 ···
69173	01157	47315	65731	67711	65359	29551	01797	47753	27795	89131 ···
19123	51107	97365	15781	17761	15309	79501	51747	97703	77745	39181 ···
14173	56157	92315	10731	12711	10359	74551	56797	92753	72795	34131 ···
19178	51152	97310	15736	17716	15354	79556	51792	97758	77790	39136 ···

Each column is a cyclic permutation of 0, 5, 5, 5, 5; 2, 7, 7, 7, 7; 4, 9, 9, 9, 9; 6, 1, 1, 1, 1; or 8, 3, 3, 3, 3. Each of these sets derives from the first set by addition of an even digit. The sum of the digits in every pentad is even, and all five pentads in a column have the same sum.

When the digits of the bracelet are written in six equal rows, the digits in each column are 60^0 apart. Thus

19673	51657	97815	15231	17211	15859	79051	51297	97253	77295	39631 ···
51039	35075	79893	99019	99851	37673	39271	99659	99299	99637	51691 ···
97411	39473	37033	39833	37695	77879	15275	93417	57091	77497	77015 ···
91437	59453	13295	95879	93899	95251	31059	59813	13857	33815	71479 ···
59071	75035	31217	11091	11259	73437	71839	11451	11811	11473	59419 ···
13699	71637	73077	71277	73415	33231	95835	17693	53019	33613	33095 ···

The digit columns are cyclic permutations of one of the four palindromes 159951, 208802, 357753 , or 406604, all of which are multiples of the first one; or of the bracelets symmetrical about a diameter 193917, 286824, 37931, or 462648, all of which are multiples of the first one. The sum of the digits in each of the pentads is even, and the sums of the pentad-digits in each column of pentads form a cyclic permutation of one of the four even sequences listed above.

The Complete 1560-Digit Bracelet

19673 51657 97815 15231 17211 15859 79051 51297 97253 77295 39631
99211 37235 77217 77239 15837 31453 35671 93035 19831 13835 95217
55853 17671 15411 17097 39877 13891 19011 13611 19235 99693 75495
31879 59037 99839 99075 13655 95431 31835 73831 57697 91639 97837
53839 33837 19077 37415 77093 91257 59677 99277 51039 35075 79893
99019 99851 37673 39271 99659 99299 99637 51691 73011 57473 15657
31677 11653 59295 51017 97477 53277 95891 31497 11877 35277 17277
39653 37819 53851 71435 35693 31633 35457 17053 53671 71659 11675
97893 77633 91695 11631 11697 35491 93259 95813 79095 37493 39495
75631 55459 31813 35233 31071 97411 39473 37033 39833 37695 77879
15275 93417 57091 77497 77015 39855 75293 93495 19493 51871 73897
71491 59497 99491 37011 91235 11079 73651 57811 77611 53097 95015
17479 77037 77453 91819 97613 77857 77677 77891 53873 19033 51219
35851 93811 33859 57675 53031 71211 59611 75473 93271 33411 95611
31611 97859 91437 59453 13295 95879 93899 95251 31059 59813 13857
33815 71479 11899 73875 33893 33871 95273 79657 75439 17075 91279
97275 15893 55257 93439 95699 93013 71233 97219 91099 97499 91875
11417 35615 79231 51073 11271 11035 97455 15679 79275 37279 53413
19471 13273 57271 77273 91033 73695 33017 19853 51433 11833 59071
75035 31217 11091 11259 73437 71839 11451 11811 11473 59419 37099
53637 95453 79433 99457 51815 59093 13633 57833 15219 79613 99233
75833 93833 71457 73291 57259 39675 75417 79477 75653 93057 57439
39451 99435 13217 33477 19415 99479 99413 75619 17851 15297 31015
73617 71615 35479 55651 79297 75877 79039 13699 71637 73077 71277
73415 33231 95835 17693 53019 33613 33095 71255 35817 17615 91617
59239 37213 39619 51613 11619 73099 19875 99031 37459 53299 33499
57013 15095 93631 33073 33657 19291 13497 33253 33433 33219 57237
91077 59891 75259 17299 77251 53435 57079 39899 51499 35637 17839
77699 15499 79499 13251'1967

★ ★ ★ ★ ★