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FIBONACCIAN ILLUSTRATION OF L'HOSPITAL'S RULE

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In [1] there is the statement: using the convention $F_0/F_0 = 1$." [$F_n = F_{n+1} + F_{n-2}$, $F_0 = 0, F_1 = 1$].

In this note it will be shown how the equation $F_0/F_0 = 1$ follows naturally from L'Hospital's Rule applied to the continuous function

$$F_x \equiv \frac{1}{\sqrt{5}} (\phi^x - \phi^{-x} \cos \pi x) \qquad [\phi = 2^{-1}(1 + \sqrt{5})] \ .$$

F_x obviously reduces to the Fibonacci numbers F_n when $n = 0, \pm 1, \pm 2, \pm 3, \cdots$. Then

$$\frac{F_0}{F_0} = \left.\frac{\frac{1}{\sqrt{5}}(\phi^x - \phi^{-x} \cos \pi x)}{\frac{1}{\sqrt{5}}(\phi^x - \phi^{-x} \cos \pi x)}\right]_{x=0} = \left.\frac{\frac{d}{dx}(\phi^x - \phi^{-x} \cos \pi x)}{\frac{d}{dx}(\phi^x - \phi^{-x} \cos \pi x)}\right]_{x=0}$$

$$= \left.\frac{(\log \phi)\phi^x - (\log \phi^{-1})\phi^{-x} \cos \pi x + \phi^{-x} \pi \sin \pi x}{(\log \phi)\phi^x - (\log \phi^{-1})\phi^{-x} \cos \pi x + \phi^{-x} \pi \sin \pi x}\right]_{x=0}$$

$$= \frac{\log \phi - \log \phi^{-1}}{\log \phi - \log \phi^{-1}} = 1 \ .$$

(Continued on p. 150.)