TRIANGLE DISSECTIONS

Mel Stover first asked [1] if it is possible to cut an obtuse triangle in smaller triangles, all of them acute. It was proven that it canbe done and that no more than seven acute triangles are necessary [2]. Martin Gardner [1] showed that a square can be dissected into no less than eight acute triangles, and then asked if a square could be dissected into less than eleven acute isosceles triangles. In the following paper by V. E. Hoggatt, Jr., and Free Jamison, the answer is given.

DISSECTION OF A SQUARE INTO n ACUTE ISOSCELES TRIANGLES

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In answer to Martin Gardner's query [3] as to whether a square can be dissected into less than eleven acute isosceles triangles, the answer is in the affirmative. We will also show that a square can be dissected into n acute isosceles triangles for $n \geq 10$.

Step 1: The 10-Piece Dissection

Dissect a square into the four triangles shown in Figure 1.

Jamison [4] applies the lemma implied by Figure 2. Thus, since triangle A may be dissected into seven acute isosceles triangles, it follows that a square may be dissected into 10 acute isosceles triangles.

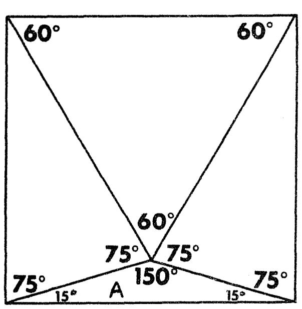

Figure 1

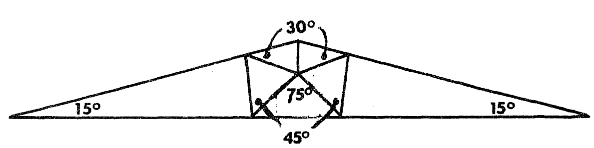

Figure 2

Step 2.

If, in Figure 3 (which is Triangle A of Figure 1), we cut off an isosceles triangle of vertex angle 15 , the remaining triangle is obtuse with $\underline{A} = 15°$, $\underline{B} = 97.5°$, and $\underline{C} = 67.5°$. In [5] it was proven that any obtuse triangle can be dissected into eight acute isosceles triangles. However, if an obtuse triangle is such that $\underline{B} > 90°$, $\underline{B} - \underline{A} < 90°$, and $\underline{B} - \underline{C} < 90°$, then only seven are needed. Thus, we can also cut a square into eleven acute isosceles triangles.

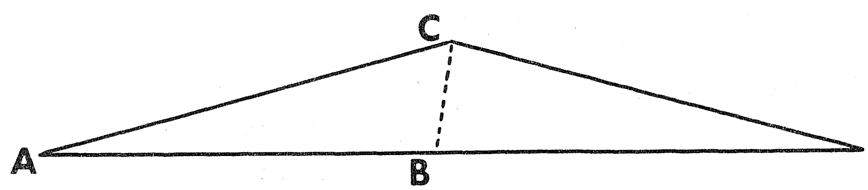

Figure 3

Step 3.

Let the triangle with angles 15°, 97.5°, and 67.5° (which can be cut into seven already) have an isosceles triangle with vertex angle 15° removed, leaving a triangle with angles 15°, 67.5°, and 97.5° which can be cut into seven acute isosceles triangles. Thus we can now cut a square into twelve acute isosceles triangles. But this last step can be repeated as many times as needed to get any $n \geq 10$ (recall we already have 10, 11, and 12). However, at the point where you had the 10-piece dissection, you can draw lines joining the midpoints of, say, the equilateral triangle (in Fig. 1) to go from 10 to 13. Then Steps 2 and 3 can go from 13 to 14 to 15. You can then cut one of the remaining equilateral triangles into four equilateral triangles.

Thus, for any large n, we may have mostly equilateral triangles if desired, or, for that matter, one of any shape as in the 10-piece dissection.

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