A FOUR-STEP ITERATION ALGORITHM TO GENERATE x in $x^2 + (x+1)^2 = y^2$

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Given $x_1 = 3$, $x_2 = 20$, $x_3 = 119$, $x_4 = 696$, and $x_5 = 4059$, we may generate all further x by the simple procedure outlined below:

$20 - 3 = 17 = 4^2 + 1$
$119 - 20 = 99 = 10^2 - 1$
$696 - 119 = 577 = 24^2 + 1$
$4059 - 696 = 3363 = 58^2 - 1$
$6 \cdot 24 = 144,\ 144 - 4 = 140,\ 140^2 + 1 = 19601,\ 19601 + 4059 = 23660 = x_6$
$6 \cdot 58 = 348,\ 348 - 10 = 338,\ 338^2 - 1 = 114243,\ 114243 + 23660 = 137903 = x_7$
$6 \cdot 140 = 840,\ 840 - 24 = 816,\ 816^2 + 1 = 665857,\ 665857 + 137903 = 803760 = x_8$
$6 \cdot 338 = 2028,\ 2028 - 58 = 1970,\ 1970^2 - 1 = 3880899,\ 3880899 + 803760 = 4684659 = x_9$

The author has taken time to check some of the newer lists against print errors. The list in [1, p. 123] should read $y_8 = 1136689$ (instead of 113689). The last column of the list in [2, p. 284] gives the first differences up to $x_{20} - x_{19}$. There are no print errors. The list in [3, p. 104] should read $x_6 = 23660$ (instead of 23360) and $x_{16} = 1070379110496$ (instead of 1070387585472), and correspondingly in the column $x + 1$ there.

REFERENCES

1. Albert H. Beiler, Recreations in the Theory of Numbers, New York, 1964.
2. Otto Emersleben, Uber zweite Binomialkoeffizienten, die Quadratzahlen sind, und Anwendung der Pellschen Gleichung auf Gitterpunktanordnungen. Wissench. Zeitschr. der Ernst-Moritz-Arndt-Universitat Greifswald, XVI (1967), pp. 279-296.
3. T. W. Forget and T. A. Larkin, "Pythagorean Triads of the Form x, x + 1, z Described by Recurrence Sequences," the Fibonacci Quarterly, Vol. 6 (June 1968), pp. 94-104.

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