

A NOTE ON FIBONACCI NUMBERS IN HIGH SCHOOL ALGEBRA

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With the number of topics in the course of study for algebra, the teacher isn't often looking for an additional unit of work, but rather for short excursions into related material to spark student interest. This note describes such a bypath.

When teaching the multiplication and division of polynomials, excellent interest-catchers are available. In multiplication, compute $(x + 1)^6$ from its definition and then display the coefficients of $(x + 1)^n$, $n = 0, 1, \dots, 6$ in the Pascal Triangle arrangement. Students readily find patterns in this array of numbers. One interesting pattern is the sums of ascending 45° diagonals — the Fibonacci sequence. Students can be asked to look for additional patterns and to report back to the class after their findings. In dividing polynomials when discussing the arrangement of terms in the divisor and remainders, let students compute a few terms of

$$x \div (1 - x - x^2) = x + x^2 + 2x^3 + \dots + F_n x^n + \dots$$

and

$$1 \div (1 - 2x) = 1 + 2x + 4x^2 + \dots + 2^n x^n + \dots$$

The teacher can easily check how the work is progressing while walking around the room because of the pattern of the quotients.

When doing computations with radicals, let students make a table with headings

$$n, \alpha^n, \beta^n, \alpha^n + \beta^n, (\alpha^n - \beta^n)/(\alpha - \beta)$$

for

$$\alpha = (1 + \sqrt{5})/2, \beta = (1 - \sqrt{5})/2, n = 1, 2, \dots, 10,$$

and then describe as many patterns observed as possible. You will be amazed at the results. Since

$$(\alpha^n - \beta^n)/(\alpha - \beta) = F_n, \quad \alpha^n + \beta^n = L_n,$$

and

$$\alpha^n = (L_n + F_n\sqrt{5})/2, \quad \beta^n = (L_n - F_n\sqrt{5})/2,$$

for the Fibonacci sequence defined by

$$F_1 = F_2 = 1, \quad F_n = F_{n-1} + F_{n-2}$$

and the Lucas sequence defined by

$$L_1 = 1, \quad L_2 = 3, \quad L_n = L_{n-1} + L_{n-2},$$

the teacher can readily check the results.

If you have found interesting uses of the Fibonacci numbers in high school teaching, you are invited to send a description to the Fibonacci Quarterly.

Continued from page 300.

SOLUTIONS TO LINEAR RECURSION RELATIONS PROBLEMS

$$1. \quad T_{n+1} = 8T_n - 18T_{n-1} + 16T_{n-2} - 5T_{n-3}$$

$$2. \quad T_n = -5/2 + 7 \times 2^n - (7/6) 3^n$$

$$3. \quad T_{n+1} = 4T_n - 2T_{n-1} - 3T_{n-2}$$

$$4. \quad T_{n+1} = 2T_n + T_{n-1} - 3T_{n-2} + T_{n-4}$$

$$(5) \quad T_n = 12 + \frac{1}{\sqrt{13}} \left(\frac{3 + \sqrt{13}}{2} \right)^n - \frac{1}{\sqrt{13}} \left(\frac{3 - \sqrt{13}}{2} \right)^n$$
