

3. Determine the recursion relation for  $T_n = P_n + Q_n$  where  $P_n$  is the arithmetic progression 3, 7, 11, 15, 19, ... and  $Q_n$  is the geometric progression 2, 6, 18, 54, ... .

4. Determine the recursion relation for  $T_n = 2^n + F_n^2$  given that the recursion relation for  $F_n^2$  is

$$F_{n+1}^2 = 2F_n^2 + 2F_{n-1}^2 - F_{n-2}^2 .$$

5. Determine the recursion relation for  $T_n = 5L_n^2 + (-1)^{n-1} + 4F_n$  .

(See page 544 for solutions to problems.)

\*\*\*\*\*

### A SHORTER PROOF

IRVING ADLER  
North Bennington, Vermont

In his article (April 1967) on 1967 as the sum of squares, Brother Brousseau proves that 1967 is not the sum of three squares. This fact can be proved more briefly as follows:

If  $1967 = a^2 + b^2 + c^2$ , where  $a$ ,  $b$  and  $c$  are positive integers, then, as Brother Brousseau has shown,  $a$ ,  $b$  and  $c$  are all odd. Then  $a = 2x + 1$ ,  $b = 2y + 1$ , and  $c = 2z + 1$ , where  $x$ ,  $y$  and  $z$  are integers.

Consequently,

$$\begin{aligned} 1967 &= (2x + 1)^2 + (2y + 1)^2 + (2z + 1)^2 \\ &= 4x^2 + 4x + 4y^2 + 4y + 4z^2 + 4z + 3. \end{aligned}$$

Then

$$1964 = 4x^2 + 4x + 4y^2 + 4y + 4z^2 + 4z .$$

Dividing by 4, we get

$$491 = x^2 + x + y^2 + y + z^2 + z ,$$

[Continued on page 551.]

\*\*\*\*\*