FIBONACCI AND THE ATOM

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Occasions of the appearance in the natural world of the Fibonacci series and of the golden section of Greek mathematics will be known to readers of this journal. In biology references, for example, the series crops up in connection with the genealogy of the drone bee, with the Nautilus sea-shell, with the florets of compositae blossoms, and in Phyllotaxis. Its appearance in the inorganic world, however, is less frequently recorded. One example is the multiple reflection of a light ray by two sheets of glass (Vol. 1, No. 1, p. 56). Another, set out below, concerns the ideally simplified atoms of a quantity of hydrogen gas.

Suppose that the single electron in one of the atoms is initially in the ground level of energy and that it gains and loses, successively, either one or two quanta of energy, so that the electron in its history occupies either the ground level (state 0) or the first energy level (state 1) or the second energy level (state 2). In this idealized case, the number of different possible histories of an atomic electron is a Fibonacci number (diagram, p. 000).

Let us make the following assumptions:

1. When the gas gains radiant energy, all state 1 atoms rise to state 2; half state zero atoms rise to state 1 and half to state 2.

2. When the gas loses energy by radiation, all the atoms in state 1 fall to state zero; half those in state 2 fall to state 1, and half to state zero.

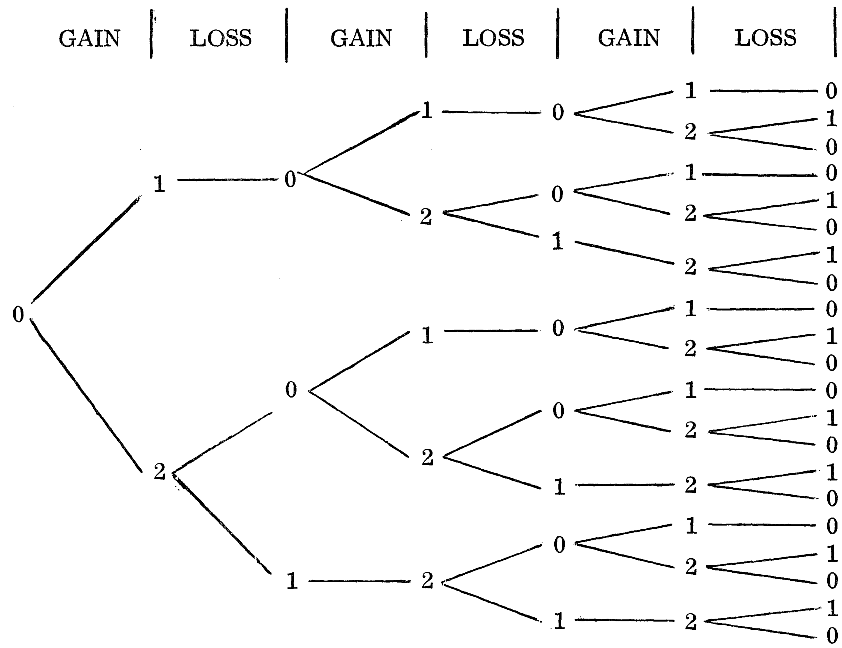
The Table shows the successive fractions of the total number of atoms found in each state. These fractions are formed exclusively of Fibonacci numbers.

A point of interest is that the fraction of atoms in the intermediate energy level (state 1) remains constant at 38.2%. If u_n is the n^{th} term of the Fibonacci series, this fraction is u_n/u_{n-1} as n tends to infinity.

$$u_n/u_{n-1} = 1 - u_{n+1}/u_{n+2} \rightarrow \varphi'^2, \quad \text{i.e., } 38.2\%.$$

The symbols φ and φ' stand for the limits of u_{n+1}/u_n and u_n/u_{n+1}, respectively as n tends to infinity. They are the roots of the equation: $x^2 - x - 1 = 0$.

POSSIBLE HISTORIES OF AN ATOMIC ELECTRON



The number of possibilities of different histories of an electron are:

		2	3	5	8	13	21 ...
Level 0	$\frac{1}{1}$	0	$\frac{2}{3}$	0	$\frac{5}{8}$	0	$\frac{13}{21} \to 0$ or $-\varphi'$
1		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{8}$	$\frac{5}{13}$	$\frac{8}{21} \to \varphi'^2$
2		$\frac{1}{2}$	0	$\frac{3}{5}$	0	$\frac{8}{13}$	$0 \to -\varphi'$ or 0

The above fractions, showing the changing proportions of atoms in each state are formed of Fibonacci numbers.

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