In the following we present a short proof of a theorem shown by R. L. Duncan [1]:

**Theorem 1.** If \( \mu_1, \mu_2, \ldots \) is the sequence of the Fibonacci numbers, then the sequence \( \log \mu_1, \log \mu_2, \ldots \) is uniformly distributed mod 1.

Moreover, we show the following proposition.

**Theorem 2.** The sequence of the integral parts \( \lfloor \log \mu_1 \rfloor, \lfloor \log \mu_2 \rfloor, \ldots \) of the logarithms of the Fibonacci numbers is uniformly distributed mod \( m \) for every positive integer \( m \geq 2 \).

**Proof of Theorem 1.** It is well known that

\[
\frac{\mu_{n+1}}{\mu_n} \to 1 + \frac{\sqrt{5}}{2},
\]

or

\[
(1) \quad \log \mu_{n+1} - \log \mu_n \to \log \frac{1 + \sqrt{5}}{2}, \quad \text{as } n \to \infty.
\]

In [2] (see th. 12.2.1), it is shown that if \( \omega \neq 0 \) is real and algebraic, then \( \theta^\omega \) is not an algebraic number. Therefore,

\[
\frac{1 + \sqrt{5}}{2}
\]

being an algebraic number, we conclude that

\[
\log \frac{1 + \sqrt{5}}{2}
\]

is transcendental. (One can also argue as follows: let be given that \( \theta > 0 \) is algebraic. Now suppose that \( \log \theta = u/v \) where \( u \) and \( v \) are integers. Then
we would have $\theta^V = e^U$. But this is impossible since $\theta^V$ is algebraic and $e^U$ is transcendental (orally communicated by A. M. Mark).

According to a theorem due to J. G. van der Corput we have that a sequence of real numbers $\lambda_1, \lambda_2, \cdots$ is uniformly distributed mod 1 if

$$\lambda_{n+1} - \lambda_n \to \theta \quad \text{(an irrational number) as } n \to \infty.$$ 

(see [3]). By the property (1) we see that the sequence $\log \mu_1, \log \mu_2, \cdots$ is uniformly distributed mod 1.

**Proof of Theorem 2.** First, we use the fact that the sequence

$$\frac{\log \mu_n}{m} \quad (m, \text{ an integer } \neq 0), \quad n = 1, 2, \cdots,$$

is uniformly distributed mod 1 which follows by the same argument used in the proof of Theorem 1: we have namely

$$\frac{\log \mu_{n+1}}{m} - \frac{\log \mu_n}{m} \to \frac{\log \frac{1 + \sqrt{5}}{2}}{m} \quad \text{(non-algebraic) as } n \to \infty.$$ 

Then according to a theorem of G. L. van den Eynden [4], quoted in [5] the sequence

$$[\log \mu_1], [\log \mu_2], \cdots$$

is uniformly distributed modulo $m$ for every integer $m \geq 2$, that is, if $A(N,j,m)$ is the number of elements of the set

$$\{[\log \mu_n] \quad (n = 1, 2, \cdots, N),$$

satisfying

$$[\log \mu_n] \equiv j \pmod{m}, \quad (0 \leq j \leq m - 1),$$

then

[Continued on page 473.]

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