Proof. Let $x = 2i\cos\theta$, $0 \le \theta \le \pi$, then from (1),

$$U_n(2i\cos\theta) = \frac{(i\cos\theta + \sin\theta)^n - (i\cos\theta - \sin\theta)^n}{2\sin\theta}$$

$$= \frac{(-i)^n(e^{-i\theta n} - e^{i\theta n})}{2\sin\theta}$$

$$U_n(2i\cos\theta) = \frac{(i)^{n-1}\sin n\theta}{\sin\theta}$$

which is zero for

$$\theta = \frac{k\pi}{n}, \qquad k = 1, 2, \cdots, n-1.$$

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[Continued from page 456.]

$$|\alpha_i|^{\frac{m+r}{r}},$$

and the circle about (1, 0) with radius $|\alpha_i|$. Now, for $\alpha_i = \alpha$, the two circles must be tangent externally (tangent, because $1 - \alpha$ is real; and externally, since $0 < 1 - \alpha < 1$). Now if there exists an i such that $|\alpha_i| < \alpha$, then the radii of both circles would be smaller, and hence they couldn't intersect. This shows that $\alpha = \alpha_i$.

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