It suffices to show that for any prime p, the highest power of p dividing the numerator is not less than that dividing the denominator. By the first part, this is equivalent to

$$(\star) \qquad \qquad \sum_{k=1}^{\infty} \left[\frac{m}{\alpha(p^{k})}\right] \geq \sum_{k=1}^{\infty} \left[\frac{r}{\alpha(p^{k})}\right] + \sum_{k=1}^{\infty} \left[\frac{m-r}{\alpha(p^{k})}\right] + \sum_{k=1}^{\infty} \left[\frac{m-r}{\alpha(p^{k})}\right]$$

But the elementary inequality $[x + y] \ge [x] + [y]$ shows that

$$\left[\frac{\mathbf{m}}{\alpha}\right] \geq \left[\frac{\mathbf{r}}{\alpha}\right] + \left[\frac{\mathbf{m} - \mathbf{r}}{\alpha}\right]$$

implying (\star) and the result.

Also solved by M. Yoder.

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[Continued from page 30.]

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