Dear Editor:

It may be of interest to your readers to note that there is a simple elementary proof of Theorem 7, page 91, Vol. 6, No. 3, June 1968, by D. A. Lind, which uses the method of descent.

To restate the Theorem,

\[5x^2 \pm 4 = y^2\]

if and only if \(x\) is a Fibonacci number and \(y\) is the corresponding Lucas number.

Proof. It is a simple identity to show that a Fibonacci and Lucas number satisfy (1) using the identities \(u_n = u_{n+1} - u_{n-1}\), \(v_n = u_{n+1} + u_{n-1}\), and \(u_n u_{n-1} - u_n^2 = (-1)^n\).

To show the converse, suppose \(x\) is the smallest positive integer which is not a Fibonacci number which satisfies (1). Then \(x \geq 4\) so that clearly \(2x < y < 3x\) and \(y\) is the same parity as \(x\). Hence, let \(y = x + 2t\) with \(t < x\). By substitution,

\[4x^2 - 4tx - 4t^2 \pm 4 = 0\]

solving for \(2x\),

\[2x = t \pm \sqrt{5t^2 \pm 4}\]

so that

\[5t^2 \pm 4 = s^2\]

where \(t\) and \(s\) are integers. Therefore \(t\) is a smaller solution to (1) than \(x\) so \(t\) must be a Fibonacci number and \(s\) is the corresponding Lucas number. But then
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2x = \( u_n \pm v_n \)

and since \( v_n > u_n \), \( n > 1 \)

2x = \( u_n + v_n = 2u_{n+1} \)

so that \( x \) is a Fibonacci number if \( t \) is, QED.

I have continued to enjoy the Fibonacci Quarterly since its inception.

Keep up the good work.

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Dear Editor:

I cheerfully donate these formulas to you. I think they have a place in the Quarterly. If you agree and feel you would like to develop a note on the basis of these formulas, I would be happy indeed.

\[
\begin{align*}
L_n &= L_n \\
L^2_n &= L_{2n} + 2(-1)^n \\
L^3_n &= L_{3n} + 3L_n(-1)^n \\
L^4_n &= L_{4n} + 4L^2_n + 2(-1)^{n+1}(-1)^n \\
L^5_n &= L_{5n} + 5L^3_n + 5L_n(-1)^{n+1}(-1)^n \\
L^6_n &= L_{6n} + 6L^4_n + 9L^2_n(-1)^{n+1} + 2(-1)^n \\
L^7_n &= L_{7n} + 7L^5_n + 14L^3_n(-1)^{n+1} + 7L_n(-1)^n \\
L^8_n &= L_{8n} + 8L^6_n + 20L^4_n(-1)^{n+1} + 16L^2_n + 2(-1)^{n+1}(-1)^n \\
L^9_n &= L_{9n} + 9L^7_n + 27L^5_n(-1)^{n+1} + 30L^3_n + 9L_n(-1)^{n+1}(-1)^n \\
L^{10}_n &= L_{10n} + 10L^8_n + 35L^6_n(-1)^{n+1} + 50L^4_n + 25L^2_n(-1)^{n+1} + 2(-1)^n \\
\end{align*}
\]

Harlan L. Umansky
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