LETTERS TO THE EDITOR

Dear Editor:

It may be of interest to your readers to note that there is a simple elementary proof of Theorem 7, page 91, Vol. 6, No. 3, June 1968, by D. A. Lind, which uses the method of descent.

To restate the Theorem,

Theorem

$$5x^2 \pm 4 = y^2 , \tag{1}$$

if and only if x is a Fibonacci number and y is the corresponding Lucas number.

Proof. It is a simple identity to show that a Fibonacci and Lucas number satisfy (1) using the identities $u_n = u_{n+1} - u_{n-1}$, $v_n = u_{n+1} + u_{n-1}$, and $u_{n+1}u_{n-1} - u_n^2 = (-1)^n$.

To show the converse, suppose x is the smallest positive integer which is not a Fibonacci number which satisfies (1). Then $x \geq 4$ so that clearly $2x < y < 3x$ and y is the same parity as x. Hence, let $y = x + 2t$ with $t < x$. By substitution,

$$4x^2 - 4tx - 4t^2 \pm 4 = 0$$

solving for $2x$,

$$2x = t \pm \sqrt{5t^2 \pm 4}$$

so that

$$5t^2 \pm 4 = s^2$$

where t and s are integers. Therefore t is a smaller solution to (1) than x so t must be a Fibonacci number and s is the corresponding Lucas number. But then

$$2x = u_n \pm v_n$$

and since $v_n > u_n$, $n > 1$

$$2x = u_n + v_n = 2u_{n+1}$$

so that x is a Fibonacci number if t is, QED.

I have continued to enjoy the Fibonacci Quarterly since its inception. Keep up the good work.

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Dear Editor:

I cheerfully donate these formulas to you. I think they have a place in the Quarterly. If you agree and feel you would like to develop a note on the basis of these formulas, I would be happy indeed.

$$L_n = L_n$$
$$L_n^2 = L_{2n} + 2(-1)^n$$
$$L_n^3 = L_{3n} + 3L_n(-1)^n$$
$$L_n^4 = L_{4n} + 4L_n^2 + 2(-1)^{n+1} \quad (-1)^n$$
$$L_n^5 = L_{5n} + 5L_n^3 + 5L_n(-1)^{n+1} \quad (-1)^n$$
$$L_n^6 = L_{6n} + 6L_n^4 + 9L_n^2(-1)^{n+1} + 2 \quad (-1)^n$$
$$L_n^7 = L_{7n} + 7L_n^5 + 14L_n^3(-1)^{n+1} + 7L_n \quad (-1)^n$$
$$L_n^8 = L_{8n} + 8L_n^6 + 20L_n^4(-1)^{n+1} + 16L_n^2 + 2(-1)^{n+1} \quad (-1)^n$$
$$L_n^9 = L_{9n} + 9L_n^7 + 27L_n^5(-1)^{n+1} + 30L_n^3 + 9L_n(-1)^{n+1} \quad (-1)^n$$
$$L_n^{10} = L_{10n} + 10L_n^8 + 35L_n^6(-1)^{n+1} + 50L_n^4 + 25L_n^2(-1)^{n+1} + 2 \quad (-1)^n$$

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