

$$\begin{aligned}
 \#(Z) &= f_n(x_1, \dots, x_n) - f_n(s_n - n + 1, 1, \dots, 1) + 1 = \\
 &= \binom{s_n}{n} - \sum_{k=1}^{n-1} \binom{s_k - 1}{k} - \left[\binom{s_n}{n} - \sum_{k=1}^{n-1} \binom{s_n - n + k - 1}{k} \right] + 1 \\
 &= - \sum_{k=1}^{n-1} \binom{s_k - 1}{k} + \sum_{k=s_n-n}^{s_n-1} \binom{s_n - k - 1}{s_n - n - 1} \\
 &= - \sum_{k=1}^{n-1} \binom{s_k - 1}{k} + \binom{s_n - 1}{s_n - n} .
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 g(x_1, \dots, x_n) &= \#(X) + \#(Y) + \#(Z) = \\
 &= 2^{\frac{s_n-1}{n}} - 1 + \sum_{k=1}^{n-1} \binom{s_n - 1}{k - 1} - \sum_{k=1}^{n-1} \binom{s_k - 1}{k} + \binom{s_n - 1}{s_n - n} \\
 &= 2^{\frac{s_n-1}{n}} - 1 + \sum_{k=1}^n \binom{s_n - 1}{k - 1} - \sum_{k=1}^{n-1} \binom{s_k - 1}{k} .
 \end{aligned}$$

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SOME RESULTS IN TRIGONOMETRY

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Graphs of the six circular functions in the first quadrant yield some particularly elegant results involving the Golden Section.

Let $\varphi^2 + \varphi = 1$, so that $\varphi = (\sqrt{5} - 1)/2 = 0.61803$ and notice that:

$$\text{arc cos } \varphi = \text{arc sin } \sqrt{1 - \varphi^2} = \text{arc sin } \sqrt{\varphi} = 0.90459$$

$$\text{arc sin } \varphi = \text{arc cos } \sqrt{1 - \varphi^2} = \text{arc cos } \sqrt{\varphi} = 0.66621$$

Further, if $\tan x = \cos x$, then $\sin x = \cos^2 x$ and $\sin^2 x + \sin x = 1$, that is, $x = \text{arc sin } \varphi$ in which case $\tan \text{arc sin } \varphi = \cos \text{arc sin } \varphi = \cos \text{arc cos } \sqrt{\varphi} = \sqrt{\varphi}$

[Continued on p. 392.]