for $k + 1 > 0$. Therefore, by Eq. (5),

$$f_n^{k+1}(a) \,|\, f_{(q+1)n}(a, f_n^{k+1-(q+1)}(a))$$

for $k + 1 \geq q + 1 > 0$, and the proof is complete.

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REFERENCES

1. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, London, 1954.
2. John Vinson, "The Relation of the Period Modulo m to the Rank of Apparition of m in the Fibonacci Sequence," Fibonacci Quarterly, Vol. 1, No. 2, April 1963, p. 38.

[Continued from page 34.]

Theorem. Let $f(x)$ be a Fibonacci function (see [1]). Then,

$$\int_1^2 f(t)dt = A \qquad (A \text{ is a constant}), \tag{2}$$

is a necessary and sufficient condition that

$$g(x) = \int_0^x f(t)dt + A, \qquad g(0) = A, \tag{3}$$

also be a Fibonacci function.

Proof. Necessity. If $g(x)$ is a Fibonacci function, then $g(x + 2) = g(x + 1) + g(x)$. For $x = 0$, $g(2) = g(1) + g(0)$, which simplifies to (2).

Sufficiency. By integration, we have

$$\int_0^x f(t + 2)dt = \int_0^x f(t + 1)dt + \int_0^x f(t)dt \ .$$

Let $t + 2 = u$ and $t + 1 = v$ to obtain

$$\int_2^{x+2} f(u)du = \int_1^{x+1} f(v)dv + \int_0^x f(t)dt \ . \tag{4}$$

Using (3), we obtain from (4), $g(x + 2) = g(x + 1) + g(x)$, by using (2).