$$F_{n+1} = \alpha F_n + A_n F_n$$
$$\alpha F_n = F_{n+1} - A_n F_n ;$$

if n is odd, from Equation (2),

$$\alpha F_n = F_{n+1} + B_n F_n .$$

These equations show that αF_n will be less than F_{n+1} if n is even, and will be greater than F_{n+1} if n is odd.

As n increases, A_n and B_n decrease fast enough that, if $n \geq 2$, $A_n F_n < 0.5$ and $B_n F_n < 0.5$.

Thus, if $n \geq 2$, it is possible to determine whether n is even or odd by multiplying F_n by α, then seeing if the product is greater than or less than the nearest integer which will be F_{n+1}. For example, given that $F_n = 21$, $21 \times 1.618 = 33.978$. This is less than the nearest integer, 34, thus a is even.

Also solved by the Proposer.

[Continued from page 437.] When X and Y are -ve integers,

$$X = (2 - L_{4k})/5, \quad Y = (X - F_{4k})/2 , \quad k = 1, 2, 3, \cdots .$$

And the general solution in +ve integers is:

$$X = (2 + L_{4k-2})/5 = F^2_{2k-1}; \quad Y = (X + F_{4k-2})/2 = F_{2k-1}F_{2k}$$

The author found the first set of integral solutions while others were found by Guy Guillotte

REFERENCES

1. J. A. H. Hunter, "Fibonacci to the Rescue," Fibonacci Quarterly, Oct. 1970, p. 406.
2. David Ferguson, "Letters to the Editor," Fibonacci Quarterly, Feb. 1970, p. 88.