

AN EXAMPLE OF FIBONACCI NUMBERS USED TO GENERATE RHYTHMIC VALUES IN MODERN MUSIC

EDWARD L. LOWMAN
200 Santa Clara Avenue, Oakland, California 94610

It has been said often enough that mathematics and music are somehow related. Whether or not such a statement can be supported in detail, proportion is certainly a major structural and expressive element in music. Time, in particular, seems to want clear-cut divisions and organization, reflecting perhaps our uneasiness in dealing with it.

In music of the twentieth century, the occurrence of two devices of temporal organization involving Fibonacci numbers stands out. One of these is the structuring of the lengths of phrases and sections in Fibonacci proportions. The other is the use of Fibonacci numbers, as well as other mathematical series and functions (even random number tables), to generate what are known as "irrational" rhythmic values.

Rhythm in Western music of the past was based on durational patterns which could be reconciled easily to some short repeating unit of time. This unit is the "beat." However long or short the beat, the rhythmic values of a passage would be multiples or divisions of it simple enough that the beat itself could be perceived.

Some composers, however, have felt that uniform beats grouped in twos, threes, and fours produced rigidity and "squareness," the so-called "tyranny of the barline." Claude Debussy, for example, deliberately obscured the beat by using long tones which did not always move at the beginning of a beat.

Not until more recent times, however, have composers tried to write irrational rhythms, rhythms which suggested no beat at all. From the outset, composers found that generating such rhythms from little numerical "games" stimulated their imaginations, assured a measure of consistency, and taught them to free their minds from old and ingrained habits. There was, after all, the present danger of falling back into beats without realizing it.

New techniques are often like this. In diatonic music the key signature appears at the beginning of every line of music, while the time signature

appears only at the beginning of the piece or at points of change. We are told that in the seventeenth century, when diatonic practice was new, musicians needed to be reminded of the key signature, while they were familiar with the older times signature. It has been suggested that the strictness of early "twelve-tone" music springs in part from a similar problem: the composers' ears were too well trained in diatonic harmony to be completely consistent in the new "atonal" medium.

Although many kinds of manipulations can produce irrational rhythms, composers have been most interested in numbers which do things. A random number table is just a bunch of numbers. A chart made from various permutations of the Fibonacci series (0, 1), a great favorite with many composers, constantly reveals surprising and provocative relationships. In the composer's mind, these are often transformed immediately into musical ideas.

The following diagram is a simple example given as an exercise by Jean-Claude Eloy (el'wah'), a prominent pupil of Pierre Boulez, when he was teaching at the Berkeley campus of the University of California. He begins with six members of the Fibonacci series (0, 1) and multiplies them by the numbers one through six so as to produce six rows of differing lengths (Figure 1).

He then scrambles the numbers one through six, or "permutes" them as he used to say, according to an arbitrarily chosen law, in this case starting with the middle two numbers and working outward by pairing the next larger with the next smaller and the largest with the smallest. The numbers 123456 now read 435261. Beside each of these numbers he now places a row from Figure 1, while the four-number row beside the number four of the new column, the three-number row beside the number three, etc. (Figure 2). These rows, too, are permuted, using the same principles, but alternating between starting at the middle and working outward and starting at the extremes and working inward. He also alternates between placing the larger number of each pair first or second. The Roman numerals represent groupings based on the position of the number six, the only one which appears three times in the array. This grouping is used at this point only to continue the alternation of permutation methods on the basis of odd- and even-numbered rows.

1	2	3	5	8	13
2	4	6	10	16	
3	6	9	15		
4	8	12			
5	10				
6					

Figure 1

4	15	3	9	6	I			
3	4	12	8			}	II	
5	16	2	10	4	6			
2	5	10				}	III	
6	13	1	8	2	5			3
1	6							

Figure 2

The rows are separated in Figure 3, and an integer is placed beneath each member. For the row containing four numbers, the numbers one through four are permuted and distributed. When all the rows have been treated in this manner, these new numbers are used to determine the number of integers to be placed at each point in yet a third row. (The permutation game continues.) When the number one appears in the second row, it simply carries down to the third to produce some long values.

Now the musical problem is posed. In each vertical group of three, the uppermost figure is to represent an amount of time, measured in seconds. The second figure represents the number of segments into which this period of time is to be divided, and the figures of the third level give the relative lengths of these segments. Thus 15/3/324 is fifteen seconds divided into three parts whose lengths can be expressed by the ratio 3:2:4. These proportions are to be written in traditional notation, with a quarter note representing one second (♩ = 60).

Row one is written out in Figure 4. As can be seen, seconds must be divided into fifths and tenths to express the three-second and nine-second units. The notation $\frac{1}{5} - 5(\frac{1}{16}) : 4 \frac{1}{7}$ means five sixteenth notes in the time of four. (Remember that a quarter note represents a second.)

I

$\overline{15''}$	$\overline{3''}$	$\overline{9''}$	$\overline{6''}$	
$\overline{3}$	$\overline{2}$	$\overline{4}$	$\overline{1}$	(1234 → 3241)
$\overline{324}$	$\overline{41}$	$\overline{1234}$		

(12345 → 34251)

II

1.	$\overline{4''}$	$\overline{12''}$	$\overline{8''}$	
	$\overline{2}$	$\overline{1}$	$\overline{3}$	(123 213)
	$\overline{23}$		$\overline{123}$	

2.	$\overline{16''}$	$\overline{2''}$	$\overline{10''}$	$\overline{4''}$	$\overline{6''}$
	$\overline{3}$	$\overline{4}$	$\overline{2}$	$\overline{5}$	$\overline{1}$
	$\overline{514}$	$\overline{2315}$	$\overline{24}$	$\overline{35142}$	

III

1.	$\overline{5''}$	$\overline{10''}$	
	$\overline{2}$	$\overline{1}$	(12 → 21)
	$\overline{21}$		

2.	$\overline{13''}$	$\overline{1''}$	$\overline{8''}$	$\overline{2''}$	$\overline{5''}$	$\overline{3''}$	
	$\overline{3}$	$\overline{4}$	$\overline{2}$	$\overline{5}$	$\overline{1}$	$\overline{6}$	(123456 → 342516)
	$\overline{516}$	$\overline{5243}$	$\overline{36}$	$\overline{61524}$		$\overline{436152}$	

3.	$\overline{6''}$
	$\overline{1}$

Figure 3

$\overline{15''}$		$\overline{3''}$		$\overline{9''}$		$\overline{6''}$
$\overline{3}$		$\overline{2}$		$\overline{4}$		$\overline{1}$
$\overline{324}$		$\overline{41}$		$\overline{1423}$		

♩ = 60

or:

Figure 4

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