script of the first Fibonacci number divisible by 41, but 41 will divide no member of the sequence between H_n and H_{n+20}.

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Perhaps the most carefully wrought example is the introduction to the first movement of the Sonata for Two Pianos and Percussion. Here the divisions, while not conforming to numbers of the Fibonacci series (0,1), are all determined by the golden mean. Measures 2–17 (the first measure is simply a roll on the timpani) contain 46 ternary (3/8) units, the most appropriate for study in a passage which contains both 6/8 and 9/8 measures. The golden mean of 46 is 28, which is the dividing line between the second and the third statements of the theme, and the place where the theme becomes inverted. The golden mean of 28 is 17.3, the juncture of the first and second statements of the theme. The two cymbal notes further subdivide the first and second statements according to the golden mean, while the third statement is so partitioned by the entrance of the tam-tam. There are other passages of the Sonata in which the phrase structure is organized according to the golden mean, but most amazing is the fact that the entire piece is so proportioned. It contains 6,432 eighth notes, and the division between the first and second movements falls but one eighth note from 3,975, the golden mean.

Will a listener be aware of the "Fibonacci proportions" as they go by? Probably not, yet they will do their job just the same. What the listener will perceive is a sense of balance, a feeling that the musical events he hears occur at the "right" places, that they form intriguing patterns in time. Composers have always played with our perception of time, causing a moment to seem interminable or a whole passage to foreshorten or "telescope" into a single recollection. To ears accustomed to fours, eights, and sixteens, these new proportions will undoubtedly seem curious in their effectiveness, but so may the phrases of a Renaissance motet or a passage of Gregorian chant.

When we analyze music, the result is a number of graphs, charts, and explanations showing how a piece achieves its effect. We must remember, however, that composers seldom create their pieces in this manner. Bartók may have had a mathematical interest in the golden mean, or he may have hit upon the Fibonacci series while consciously searching for shapes other than powers of two for his musical ideans. More likely, however, the techniques grew out of the shapes of the musical ideas themselves, just as have most new techniques throughout music history. One can imagine his realizing at some point that these proportions were what his ideas had been approaching all along. The technique was thus a means of focusing and clarifying the effect. Whatever the procedure Bartók used, we know that performers and listeners recognized the exceptional balance and proportion of this music long before anyone discovered its "astonishing" use of Fibonacci numbers and the golden mean.

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