THE SUM OF THE FIRST n POSITIVE INTEGERS—GEOMETRICALLY

FREDERICK STERN
San Jose State College, San Jose, California

The familiar formula $1 + 2 + \cdots + n = \frac{1}{2}n(n + 1)$ follows from counting in two ways, the number of intersections of $(n + 1)$ lines in the plane, assuming that no two of these lines are parallel and no three intersect at the same point. On the one hand, since any two of the lines intersect at a point distinct from the point of intersection of any other pair, the number of points of intersection is the same as the number of distinct pairs of lines:

$$\binom{n+1}{2} = \frac{1}{2}n(n + 1) \ .$$

On the other hand, suppose the lines are numbered, completely arbitrarily, from 1 to $(n + 1)$. Counting the number of intersections sequentially, the second line intersects the first at one point. The third line intersects each of the first two at two distinct points — giving a partial total of $1 + 2$ intersections. The fourth line intersects each of the first 3 at three points — giving a partial total of $1 + 2 + 3$ intersections. Thus, the $(k + 1)^{st}$ line intersects the first through the k^{th} lines at k distinct points so that the lines numbered 1 through $(k + 1)$ intersect at $1 + 2 + \cdots + k$ distinct points. Finally, we see in this way that the $n + 1$ lines intersect at $1 + 2 + \cdots + n$ distinct points. Thus, we have counted the same number of points in two ways and have arrived at the familiar formula.

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Afternoon Session

Phyllotaxis: The Facts and the Theory
Dr. Irving Adler, North Bennington, Vermont

Telephone Grammars: An Elementary Example in the Mathematical Theory of Context-Free Languages
George Ledin, Jr., Institute of Chemical Biology, University of S. F.

The Periodic Properties of a Linear Recurrent Sequence over a Ring
Prof. Donald W. Robinson, Brigham Young University, Provo, Utah

Free Discussion Period